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NO. 1267

ESTIMATION OF EXTRA DRAG FORCE ON A CIRCULAR CYLINDER DUE TO THE PRESENCE OF SOLID PARTICLES IN SUBSONIC COMPRESSIBLE FLOW

by

Stanley B. Mellsen

PCN 051SA

August 1989

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ABSTRACT

The effect of particles, such as dust in air, on aerodynamic drag of circular cylinders was calculated for compressible flow at critical Mach number and for incompressible flow. The effect of compressibility was found negligible for particles larger than about 10 μm, for which the air can be considered a continuum. Drag coefficient and collection efficiency are provided for a wide range of inertia parameters and Reynolds numbers for both compressible and incompressible flow.

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NOTATION

d	particle diameter, cm
E_m	impaction efficiency of particles on a target
F_p	force exerted on the cylinder by the particles alone travelling in air, dynes
L	cylinder radius, cm
p	drag pressure on cylinder due to stream of particles alone, dynes cm^{-2}
r	distance from any point to the origin, cm
t	time, seconds
U	free-stream velocity, cm s^{-1}
u	local fluid velocity, cm s^{-1}
u_x	parallel component of fluid velocity, cm s^{-1}
u_y	transverse component of fluid velocity, cm s^{-1}
v	local particle velocity, cm s^{-1}
v_x	component of particle velocity parallel to free field flow direction immediately before impact, cm s^{-1}
v_x'	component of particle velocity parallel to flow direction immediately after impact and reflection, cm s^{-1}

v_{xo}	far upstream particle velocity, cm s^{-1}
v_y	component of particle velocity perpendicular to free field flow direction immediately before impact, cm s^{-1}
Δv_x	change in the component of particle velocity parallel to the free stream flow direction caused by reflection from the sphere or cylinder, cm s^{-1}
x	co-ordinate (origin at centre of sphere or cylinder) of particle position parallel to free stream flow direction, cm
y	transverse co-ordinate of particle position, cm
α	angle of incidence of a particle against the sphere or cylinder, radians
β	angle between particle direction and free field flow direction just before the particle hits the sphere or cylinder, radians
γ	total angle change of a particle from its far upstream direction to its direction after reflection from the cylinder or sphere, radians
θ	polar angle between negative x axis and radius vector to particle impact position, radians
λ	mean free path of molecules μm
μ	local absolute viscosity of fluid, poise
μ_o	free stream absolute viscosity of fluid, poise

ρ local fluid density, g cm^{-3}

ρ_0 fluid density at free stream Mach number g cm^{-3}

ρ_S fluid density of still air g cm^{-3}

ρ_P uniform particle mass per unit volume of air far upstream, g cm^{-3}

σ particle density g cm^{-3}

ω angle between positive x axis and position in flow field radians

The following were dimensionless:

C_C Cunningham Correction Factor

C_D drag coefficient due to particle impact with cylinder

$f(\bar{y})$ change in component of particle velocity due to reflection from sphere or cylinder as a function of off-axis distance far upstream

K particle inertia parameter

M Mach number

Re spherical particle Reynolds number in flow influenced by presence of cylinder

Re_0 spherical particle Reynolds number in free stream

$\bar{r}$ — $\frac{r}{L}$, distance from any point to the origin

T — time $\frac{tU}{L}$

$\bar{u}$ — local fluid velocity $\frac{u}{U}$, $\bar{u}_x$ and $\bar{u}_y$ are x and y components

$\bar{v}$ — local particle velocity $\frac{v}{U}$, $\bar{v}_x$ and $\bar{v}_y$ are x and y components

$\bar{v}_x$ — $\frac{d\bar{x}}{dT}$, parallel component of particle velocity

$\bar{v}_y$ — $\frac{d\bar{y}}{dT}$, transverse component of particle velocity

$\bar{x}$ — parallel co-ordinate $\frac{x}{L}$

$\bar{y}$ — transverse co-ordinate $\frac{y}{L}$

$\bar{y}'$ — off-axis distance of a particle at point of impact with the cylinder

$\bar{y}_d$ — far upstream transverse co-ordinate of the envelope of particles which eventually hit the cylinder

$\bar{y}_i$ — co-ordinate of particle used in discretization for integration; $\bar{y}_i$ increases with i from $\bar{y}_1 = o$ to $\bar{y}_n = \bar{y}_d$

γ_a — specific heat ratio for air

ϕ — dimensionless group independent of drop size formed by combining Re_0 and K

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11. Comparison of Present Calculation Method to Theory and Experiment of Other Investigators

INTRODUCTION

Extensive studies have been made of the drag exerted on transverse cylinders in low speed steady flow of moving air for which compressibility effects are negligible [1]. These results are shown for Reynolds numbers up to 10^6, which barely extends into the supercritical Reynolds number region. Further work with large cylinders in a wind tunnel, with velocities low enough so that compressibility effects are still negligible, have provided information about aerodynamic drag reaching further into the supercritical region with Reynolds numbers from 10^6 to 10^7 [2]. Experiments in which the flow velocities are high enough to provide an effect on drag of cylinders due to compressibility have also been performed [3]. Similar information has been provided for Spheres [1]. Some information is also available for unsteady incompressible flow which was obtained from field experiments with large blast waves [4, 5]. The reports cited generally show that drag coefficient increases with Mach number for Mach numbers between 0.3 and 1.

The blast waves from 500 ton TNT bursts on the DRES test range were associated with visible quantities of dust picked up from the dry prairie ground. This was observed by the author during Operation Prairie Flat in 1968 and in Event Dial Pack in 1970. Therefore, to fully understand the forces on targets in such a blast wave, a knowledge of the effect of the dust is necessary. Attempts with laboratory experiments have been performed to determine the effect of introducing dust into the air stream [6, 7] and to calculate the effect using a mathematical model [8]. Only incompressible flow was considered in these experimental and theoretical studies.

The purpose of this report is to describe numerical solutions to the equations of motion of particles moving toward and impacting with a transverse circular cylinder in compressible air flow. The solutions are subsequently applied to calculate drag coefficients due to particles alone and the significance of compressibility is described.

DEFINITION OF THE PROBLEM

A cloud of spherical particles uniformly distributed in steady air flow is assumed to travel toward a circular cylinder with its axis perpendicular to the flow direction. Far upstream, the particles are assumed to travel along with the velocity and direction of the air, but as they approach the cylinder they will not necessarily follow streamlines because the air velocity changes in both speed and direction in the vicinity of the cylinder. Far upstream, the particles which travel toward the cylinder and eventually contact it will be contained within a width less than the cylinder diameter. This distance or limiting envelope divided by the cylinder diameter is called the collection efficiency. The velocity of a particle immediately before contact and also the point of contact with the cylinder depend upon the position of

the particle within the limiting envelope far upstream from which it travels to the target. All particles have their momentum changed when they collide with the cylinder and, in the case of solid particles, are deflected in a new direction. The total change in momentum of all particles which hit the cylinder is balanced by the force they exert on it. The basic problem is, therefore, to calculate the collection efficiency and then to calculate the force of particles on the cylinder, from which the drag coefficient due to the particles alone can be obtained.

EQUATIONS OF MOTION

The motion of an individual spherical particle has been shown [9] to be determined by the following ordinary differential equations:

$$\frac{d\bar{v}_y}{dT} = \frac{C_D Re(\bar{u}_y - \bar{v}_y)}{24K} \tag{1}$$

$$\frac{d\bar{v}_x}{dT} = \frac{C_D Re(\bar{u}_x - \bar{v}_x)}{24K} \tag{2}$$

where

$$Re = Re_0 \left[(\bar{u}_y - \bar{v}_y)^2 + (\bar{u}_x - \bar{v}_x)^2\right]^{\frac{1}{2}} \tag{3}$$

$$K = \frac{\sigma d^2 U}{18\mu_0 L} \text{ particle inertia parameter} \tag{4}$$

$$Re_0 = \frac{Ud\rho_0}{\mu_0} \text{ free stream Reynolds number} \tag{5}$$

The symbols are defined in the Notation Section near the front of this report and the basic geometry of the flow system is illustrated in Fig. 1.

The above equations of motion are for incompressible flow. In compressible flow the density changes as flow velocity changes near the cylinder. Then equation (3) must be replaced by the following equation:

$$Re = Re_0\left(\frac{\rho}{\rho_0}\frac{\mu_0}{\mu}\right)[(\bar{u}_y - \bar{v}_y)^2 + (\bar{u}_x - \bar{v}_x)^2]^{\frac{1}{2}} \tag{6}$$

The density ratio ρ/ρ_0 is calculated from the energy equation for isentropic flow [10] which is written as follows:

$$\frac{\rho_s}{\rho_0} = \left(1 + \frac{\gamma_a - 1}{2} M_0^2\right)^{\frac{1}{\gamma_a - 1}} \tag{7}$$

where ρ_s is for $M_0 = 0$ and ρ_0 is for free stream Mach number, M_0.

Also
$$\frac{\rho_s}{\rho} = \left(1 + \frac{\gamma_a - 1}{2} M^2\right)^{\frac{1}{\gamma_a - 1}} \tag{8}$$

where ρ is for the local Mach number M, near the cylinder which is given by the following equation:

$$M = M_0\sqrt{\bar{u}_x^2 + \bar{u}_y^2} \tag{9}$$

Then by combining equations (7), (8) and (9), the ratio of densities in equation (6) is obtained in terms of the free stream Mach number as

follows:

$$\frac{\rho}{\rho_0} = \left[\frac{1 + \frac{1}{2}(\gamma_a - 1) M_0^2}{1 + \frac{1}{2}(\gamma_a - 1) M_0^2 (\bar{u}_x^2 + \bar{u}_y^2)}\right]^{\frac{1}{\gamma_a - 1}} \tag{10}$$

Several assumptions were inherent in the development of equation (1) and equation (2) for calculating impaction efficiency and force due to a stream of particles, including:

a. uniform particle distribution;

b. no gravitational or electrostatic forces of consequence;

c. monodisperse spherical particles with diameter negligible in relation to the target sphere or cylinder diameter; and

d. free stream flow that was steady and irrotational;

e. the ratio of the local viscosity μ to the free stream viscosity μ_0, is unity. This assumption is valid, because viscosity is not appreciably affected by the order of magnitude of the changes in temperature found along a trajectory.

The drag coefficient is a function of Reynolds number and was available in the form of definitive empirical equations [11]. These equations are stated as follows:

$$Re = \frac{C_D Re^2}{24} - 2.3363 \times 10^{-4}(C_D Re^2)^2 + 2.0154 \times 10^{-6}(C_D Re^2)^3 - 6.9105 \times 10^{-9}(C_D Re^2)^4 \tag{11}$$

for $Re < 4$ or $C_D Re^2 < 140$

$$\log_{10} Re = -1.29536 + 9.86 \times 10^{-1} (\log_{10} C_D Re^2) - 4.6677 \times 10^{-2}(\log_{10} C_D Re^2)^2 + 1.1235 \times 10^{-3} (\log_{10} C_D Re^2)^3 \tag{12}$$

for $3 < Re < 10^4$ or $100 < C_D Re^2 < 4.5 \times 10^7$

Equation (11) approaches Stokes flow as the Reynolds number approaches zero. These equations assume that the fluid is a continuum. For Knudsen number, which is the ratio of the molecular mean free path in the fluid to the diameter of the particle, less than a value of the order of 10^{-3}, the particle is large by comparison with the scale of molecular processes in the continuous phase and the fluid can be treated as a continuum. The mean free path for air, at 293 K temperature, and pressure of 100 kP, is 0.0653 µm. Thus, noncontinuous effects are significant for particles smaller than about 10 µm [12]. The actual significance on collection efficiency and drag coefficient was estimated by dividing the drag coefficient given by equations (11) and (12) by the Cunningham correction factor and calculating the effects for some of the smaller particle sizes considered. The factor is applied to account for the fact that the fluid resistance is smaller than that given by Stokes law when noncontinuum effects are present. The correction factor is written as follows [13]:

$$C_c = 1 + \frac{2\lambda}{d}\left[1.257 + 0.4 \exp\left(\frac{-1.1d}{2\lambda}\right)\right] \tag{13}$$

AIR FLOW FIELD

The compressible-air flow field around a right circular cylinder of infinite extent for flow perpendicular to the cylinder axis is

described by the following equations [14]:

$$
\begin{aligned}
\bar{u}_x = {} & [(1 - \bar{r}^{-2}) \cos^2\omega + (1 + \bar{r}^{-2}) \sin^2\omega] + \\
& M^2 \left[\left(-\frac{13}{12}\bar{r}^{-2} + \frac{3}{2}\bar{r}^{-4} - \frac{5}{12}\bar{r}^{-6}\right) \cos^2 \omega \left(\frac{1}{4}\bar{r}^{-2}\ \frac{1}{4}\bar{r}^{-4}\right) \cos \omega \cos 3 \omega - \right. \\
& \left. \left(-\frac{13}{12}\bar{r}^{-2} + \frac{1}{2}\bar{r}^{-4} - \frac{1}{12}\bar{r}^{-6}\right) \sin^2 \omega - \left(\frac{3}{4}\bar{r}^{-2} - \frac{1}{4}\bar{r}^{-4}\right) \sin \omega \sin 3 \omega \right] + \\
& M^4 \left\{ (\gamma-1) \left[\left(-\frac{17}{60}\bar{r}^{-2} + \frac{3}{8}\bar{r}^{-4} - \frac{5}{12}\bar{r}^{-6} + \frac{7}{16}\bar{r}^{-8} - \frac{9}{80}\bar{r}^{-10}\right) \cos^2 \omega + \right.\right. \\
& \left. \left(+\frac{61}{80}\bar{r}^{-4} - \frac{15}{16}\bar{r}^{-6} + \frac{7}{40}\bar{r}^{-8}\right) \cos \omega \cos 3 \omega \left(-\frac{3}{16}\bar{r}^{-4} + \frac{3}{16}\bar{r}^{-6}\right) \cos \omega \cos 5 \omega \right] + \\
& \left[\left(-\frac{137}{80}\bar{r}^{-2} + 4\bar{r}^{-4} - \frac{65}{16}\bar{r}^{-6} + \frac{35}{16}\bar{r}^{-8} - \frac{33}{80}\bar{r}^{-10}\right) \cos^2 \omega + \right. \\
& \left(+\frac{19}{48}\bar{r}^{-2} + \frac{5}{16}\bar{r}^{-4} - \frac{15}{16}\bar{r}^{-6} + \frac{7}{24}\bar{r}^{-8} - \frac{1}{16}\bar{r}^{-10}\right) \cos \omega \cos 3 \omega + \\
& \left. \left(-\frac{1}{16}\bar{r}^{-2} - \frac{3}{16}\bar{r}^{-4} + \frac{1}{4}\bar{r}^{-6}\right) \cos \omega \cos 5 \omega \right] - \\
& (\gamma-1) \left[\left(-\frac{17}{60}\bar{r}^{-2} + \frac{1}{8}\bar{r}^{-4} - \frac{1}{12}\bar{r}^{-6} + \frac{1}{16}\bar{r}^{-8} - \frac{1}{80}\bar{r}^{-10}\right) \sin^2 \omega + \right. \\
& \left. \left(\frac{61}{80}\bar{r}^{-4} - \frac{9}{16}\bar{r}^{-6} + \frac{3}{40}\bar{r}^{-8}\right) \sin \omega \sin 3 \omega + \left(-\frac{5}{16}\bar{r}^{-4} + \frac{3}{16}\bar{r}^{-6}\right) \sin \omega \sin 5 \omega \right] - \\
& \left[\left(-\frac{137}{80}\bar{r}^{-2} + \frac{4}{3}\bar{r}^{-4} - \frac{13}{16}\bar{r}^{-6} + \frac{5}{16}\bar{r}^{-8} - \frac{11}{240}\bar{r}^{-10}\right) \sin^2 \omega + \right. \\
& \left(\frac{19}{16}\bar{r}^{-2} + \frac{5}{16}\bar{r}^{-4} - \frac{9}{16}\bar{r}^{-6} + \frac{1}{8}\bar{r}^{-8} - \frac{1}{48}\bar{r}^{-10}\right) \sin \omega \sin 3 \omega + \\
& \left. \left. \left(-\frac{5}{16}\bar{r}^{-2} - \frac{5}{16}\bar{r}^{-4} + \frac{1}{4}\bar{r}^{-6}\right) \sin \omega \sin 5 \omega \right] \right\}
\end{aligned}
\qquad (14a)
$$

and

$$
\begin{aligned}
\bar{u}_y = {} & [-\bar{r}^{-2} \sin 2\omega] + \\
& M^2 \Bigg[\bigg(-\frac{13}{12}\bar{r}^{-2} + \frac{3}{2}\bar{r}^{-4} - \frac{5}{12}\bar{r}^{-6}\bigg) \sin\omega \cos\omega \quad \frac{1}{4}\bigg(-\bar{r}^{-2} - \frac{1}{4}\bar{r}^{-4}\bigg) \sin\omega \cos 3\omega + \\
& \bigg(-\frac{13}{12}\bar{r}^{-2} + \frac{1}{2}\bar{r}^{-4} - \frac{1}{12}\bar{r}^{-6}\bigg) \sin\omega \cos\omega + \bigg(\frac{3}{4}\bar{r}^{-2} - \frac{1}{4}\bar{r}^{-4}\bigg) \cos\omega \sin 3\omega\Bigg] + \\
& M^4 \Bigg\{(\gamma-1)\Bigg[\Bigg[\bigg(-\frac{17}{60}\bar{r}^{-2} + \frac{3}{8}\bar{r}^{-4} - \frac{5}{12}\bar{r}^{-6} + \frac{7}{16}\bar{r}^{-8} - \frac{9}{80}\bar{r}^{-10}\bigg) \sin\omega \cos\omega + \\
& \bigg(\frac{61}{80}\bar{r}^{-4} - \frac{15}{16}\bar{r}^{-6} + \frac{7}{40}\bar{r}^{-8}\bigg) \sin\omega \cos 3\omega + \bigg(-\frac{3}{16}\bar{r}^{-4} + \frac{3}{16}\bar{r}^{-6}\bigg) \sin\omega \cos 5\omega\Bigg] + \\
& \Bigg[\bigg(-\frac{137}{80}\bar{r}^{-2} + 4\, r^{-4} - \frac{65}{16}\bar{r}^{-6} + \frac{35}{16}\bar{r}^{-8} - \frac{33}{80}\bar{r}^{-10}\bigg) \sin\omega \cos\omega + \\
& \bigg(\frac{19}{48}\bar{r}^{-2} + \frac{5}{16}\bar{r}^{-4} - \frac{15}{16}\bar{r}^{-6} + \frac{7}{24}\bar{r}^{-8} - \frac{1}{16}\bar{r}^{-10}\bigg) \sin\omega \cos 3\omega + \\
& \bigg(-\frac{1}{16}\bar{r}^{-2} - \frac{3}{16}\bar{r}^{-4} + \frac{1}{4}\bar{r}^{-6}\bigg) \sin\omega \cos 5\omega\Bigg] + \\
& (\gamma-1)\Bigg[\Bigg[\bigg(-\frac{17}{60}\bar{r}^{-2} + \frac{1}{8}\bar{r}^{-4} - \frac{1}{12}\bar{r}^{-6} + \frac{1}{16}\bar{r}^{-8} - \frac{1}{80}\bar{r}^{-10}\bigg) \sin\omega \cos\omega + \\
& \bigg(\frac{61}{80}\bar{r}^{-4} - \frac{9}{16}\bar{r}^{-6} + \frac{3}{40}\bar{r}^{-8}\bigg) \sin 3\omega \cos\omega + \bigg(-\frac{5}{16}\bar{r}^{-4} + \frac{3}{16}\bar{r}^{-6}\bigg) \sin 5\omega \cos\omega\Bigg] + \\
& \Bigg[\bigg(-\frac{137}{80}\bar{r}^{-2} + \frac{4}{3}\bar{r}^{-4} - \frac{13}{16}\bar{r}^{-6} + \frac{5}{16}\bar{r}^{-8} - \frac{11}{240}\bar{r}^{-10}\bigg) \sin\omega \cos\omega + \\
& \bigg(\frac{19}{16}\bar{r}^{-2} + \frac{5}{16}\bar{r}^{-4} - \frac{9}{16}\bar{r}^{-6} + \frac{1}{8}\bar{r}^{-8} - \frac{1}{48}\bar{r}^{-10}\bigg) \sin 3\omega \cos\omega + \\
& \bigg(-\frac{5}{16}\bar{r}^{-2} - \frac{5}{16}\bar{r}^{-4} + \frac{1}{4}\bar{r}^{-6}\bigg) \sin 5\omega \cos\omega\Bigg]\Bigg\}
\end{aligned}
\tag{14b}
$$

The first bracketed term describes the incompressible flow around the cylinder. The term with the M^2 factor and the term with the M^4 factor are, respectively, the first and second order corrections for the effect of compressibility. In equations (14a, b) ω is measured counterclockwise from the x - axis of the cylinder as is shown in Figure 1. Equations (14a, b) are not the same as the equations stated in reference [14], but close inspection reveals that the difference is due only to an error in writing the equations in the reference. Equations (14a, b) were derived from a velocity potential function [15].

DRAG FORCE DUE TO PARTICLES ALONE

The force F_p of a stream of particles hitting a cylinder is calculated from the change in momentum. Assuming perfectly elastic spherical particles, that the particles do not interact with each other, and that only one impaction of each particle occurs, or at least is significant, the drag force is given by the following equation:

$$\frac{F_p}{\rho_p U^2 L} = 2\bar{v}_{xo} \int_0^{\bar{y}_d} \Delta\bar{v}_x d\bar{y} \tag{15}$$

$\Delta\bar{v}_x$ is the change of the particle velocity component in the direction of free stream flow due to reflection from the cylinder. It is given by:

$$\Delta\bar{v}_x = \bar{v}_x - \bar{v} \cos \gamma \tag{16}$$

where $$\bar{v} = \sqrt{\bar{v}_x^{\,2} + \bar{v}_y^{\,2}} \tag{17}$$

and γ is the angle the reflected particle makes with the free stream flow direction as shown in Figure 2.

$$\gamma = \pi - 2\tan^{-1}\left(\frac{\bar{y}'}{\sqrt{1-\bar{y}'^2}}\right) - \tan^{-1}\left(\frac{\bar{v}_y}{\bar{v}_x}\right) \tag{18}$$

The drag coefficient is defined as follows:

$$C_D = \frac{p}{\frac{1}{2}\rho_p U^2} \tag{19}$$

where p is the average drag pressure [1].

Then, using equations (15), (16), and (19), the drag coefficient is given by the following equation:

$$C_D = 2\bar{v}_{xo} \int_0^{\bar{y}_d} (\bar{v}_x - \bar{v}\cos\gamma)\,d\bar{y} \tag{20}$$

A more detailed derivation of equation 20 is shown elsewhere [8].

Equation 20 was solved numerically by dividing the upstream cross sectional area into strips of equal width Δy, and calculating the contribution from each strip. These contributions were then summed as follows:

$$\text{Let} \quad \bar{v}_x - \bar{v}\cos\gamma = f(\bar{y}) \tag{21}$$

Then the integral in equation (20) is written:

$$\int_{o}^{\bar{y}_d} f(\bar{y})d\bar{y} \sim \frac{f(\bar{y}_1) + f(\bar{y}_n)}{2}\Delta\bar{y} + \sum_{i=2}^{i=n-1} f(\bar{y}_i)\Delta\bar{y} \qquad (22)$$

where $\bar{y}_i$ increases with i from $\bar{y}_1 = o$ to $\bar{y}_n = \bar{y}_d$.

To solve equation (22) to obtain the drag coefficient given by equation (20) it is necessary to know the values of $\bar{v}_x$, $\bar{v}$, and γ in the range $0 \leq \bar{y} \leq \bar{y}_d$.

The first step was to find the value of $\bar{y}_d$, the impaction efficiency, which was the upper limit of the integral. This was done by an iterative procedure called the half interval method [16]. The value of $\bar{y}$ for the critical particle was estimated far upstream and the path followed to the target. The difference between the ordinate of the tangent path and the ordinate of the point on the actual path parallel to the tangent path was the miss criterion used. The direction of the tangent path was not known a priori but was assumed parallel to the actual path. The half interval method previously mentioned was applied to determine a better initial estimate. Then the path was followed to the target again for another calculation of miss distance. This process was repeated several times until convergence was achieved. The plane of initial position of particles, which was perpendicular to the flow direction, was located far enough from the target so that free stream conditions prevailed. A distance of 20 target radii upstream of the target centre was found to be sufficient. The particles were assumed to travel with the speed and direction of the fluid at this upstream position. Starting calculations further upstream did not change results significantly.

The path of an individual particle was determined step by step by applying a fourth order Runge-Kutta method [16] to the equations of motion (1) and (2). The value K in these equations was determined initially, once and for all, for the whole calculation from equation (4) since it did not change during the procedure. The value of Re was calculated for each time step using equation (6) with values from the previous time step and equation (5) in which Re_0 was constant. The value of $C_D Re$ was calculated in the same manner by numerical solution of equations (11) and (12) using Newton's method [16] for finding the zeros of a function. The values of $\bar{u}_x$ and $\bar{u}_y$ were calculated using equations (14a, b).

Once the value of $\bar{y}_d$ had been determined, the integration procedure of equation (22) was applied and the drag coefficient was calculated from equation (20). The values of $f(\bar{y})$ used in equation (22) were determined from equation (20) for each particle path by tracing a particle from its initial position to the cylindrical target and using the velocities just before impaction in equacions (16) and (17) along with the ordinate of the impaction point in equation (18).

Computer programs were written in Fortran for the DRES Honeywell DP8/70C using the CP-6 operating system to calculate impaction efficiency and drag coefficient for various particle and cylinder diameters. The complete program and one set of results is shown in Annex A. The application of the method is straightforward, with the exception of the step by step integration procedure near the target cylinder. This procedure is therefore explained as follows. The particle was calculated in time steps, ΔT, until the particle was one time step or less away from the target. Then the time increment size was decreased by a factor of ten and step by step integration continued until the particle was one new time step or less away from the target. Finally, the time increment size was decreased by another factor of ten

and the integration allowed to proceed to the target again. This method ensured that the position of the target and particle coincided within a maximum error given by the distance travelled during one percent of the original time step size, while allowing the particle to reach the proximity of the target in an adequate but small number of steps, to use computing time efficiently.

RESULTS

The same particle tracing technique was used in obtaining the collection efficiency and drag coefficient. Therefore, a comparison of the calculated results to existing information on collection efficiency, for which previous information is available, is helpful to gain confidence in the results. Collection efficiencies were calculated for a wide range of inertia parameters with particle specific gravity one (Table 1) and plotted in Figure 3 along with results obtained by a mechanical analogue [14], which was fundamentally a differential analyzer constructed for solving the equations of motion [17]. The collection efficiencies were compared for a free stream Mach number of 0.4 and a value of ϕ of 1000, where ϕ is a dimensionless parameter independent of particle size given by the following equation:

$$\phi = \frac{Re_o^{\,2}}{K} \tag{23}$$

The air properties used were obtained from the ICAO Standard Atmosphere at an altitude of 3048 m (10000 ft) [18, 19]. The compressible-air velocity components in the flow field from equation (14a, b) were calculated at a Mach number of 0.4 because the effect of compressibility for completely subsonic flow is considered greatest at or near the free stream critical Mach number [14]. The value obtained for the free stream critical Mach number depends on the number of correction terms applied to the incompressible flow field and appears to approach 0.4 as

a limit [15]. The value of 1000 for ϕ was chosen as a representative middle value from the results reported in reference [14].

Collection efficiencies were also calculated for incompressible flow velocity using the same values of K, ϕ and atmospheric conditions as for the compressible flow. This was accomplished simply by setting M = 0 in equations (14a,b) and performing the calculations. The collection efficiencies, along with the drag coefficients for both compressible flow at Mach number 0.4 and incompressible flow are shown in Table 1. The collection efficiencies for incompressible flow, are also plotted in Figure 4, along with results obtained from the mechanical analogue previously described [17, 21].

Calculation of collection efficiencies and drag coefficients for a wide range of inertia parameters, using particle specific gravity of 2.6, the density of sand, and dimensionless parameters, ϕ, were performed using the ICAO standard atmosphere at sea level (Table 2). These are intended mainly to provide information to engineers who are interested in blast hardening of structures, but they could also be applied to estimate collection efficiencies of drops or solid particulates in chemical defence problems. The calculated results of collection efficiencies are shown along with fitted curves for Mach number 0.4, and incompressible flow, in Figures 5 and 6, respectively. Similarily, the drag coefficients are shown in Figures 7 and 8.

An example of the distribution of forces for elastic reflection of particles from a cylinder is shown in Figure 9, where the function $\bar{v}_{xo}(\bar{v}_x - \bar{v}\cos\gamma)$ is plotted as a function of the perpendicular distance from a line through the center of the cylinder along the free stream flow direction. Comparison of surface velocity on a cylinder between an incompressible flow field and a compressible-air flow field

at Mach number 0.4 is shown in Figure 10 [14].

DISCUSSION

The calculated collection efficiencies agree favourably with the theoretical results of other workers for which a mechanical analogue was used, as can be seen in Figures 3 and 4. The difference in results may be at least partly explained by the fact that the equations of motion, (1) and (2), were considered mathematically in two different ways. The other workers [14, 17] used a linearized approximation for the equation of motion between $\bar{x} = -\infty$ and $\bar{x} = -5$. The equations were then integrated directly from $\bar{x} = -5$ to the cylinder using the starting conditions at $\bar{x} = -0.5$ obtained from the linearized equations [17]. The present method integrates the equations of motion directly from $\bar{x}$ = -20.0 to the cylinder. The results agree very closely for the highest values of the inertia parameter, but differ more for the smaller values. Experimenting with various time step sizes showed that the results were sensitive to variation in this parameter. All the results shown in Figures 3 to 8 were calculated with a time step size of ΔT = 0.1, which was chosen such that convergence was provided while keeping computation time reasonably short. Using this time step size, one data point could be obtained in 30 minutes from an IDM RESEARCH T286 operating at 11.7 mega-Herz, with math co-processor, while 6 minutes were required with the DRES Honeywell DP8/70C and 1 minute with the DRES FPS array processor. The effect of reducing ΔT by a factor of 5 is shown for one value of ϕ and seven values of K in Table 3. The differences due to the time step difference in the collection efficiencies and drag coefficients over the whole range of inertia parameters are less than 0.026 and 0.061, respectively. The percentage errors increase with decreasing values of K due to the fact that the absolute values decrease with K, as can also be seen in Table 3. The effect of varying time step size over a wide range is shown in Table 4 for the

smallest value of K considered. The results indicate that the value of the calculated point is very nearly a linear function of time step size for both collection efficiency and drag coefficient. The intercept for $\Delta T = 0$ is 0.0634 for collection efficiency, which falls slightly below the results presented in reference [14], as can be seen by comparison to the curve plotted in Figure 3. The fluid velocity components given by the long equations (14a, b) were plotted as shown in Figure 10. The calculated results shown agree with those in references [14] and [15], which contain identical figures, as a comparison check showed.

There is some experimental data available for comparison to calculated theoretical results. Measurements of the collection efficiency of cylinders by Ranz and Wong [22] represent the main body of data on which comparisons of theory and experiment are based [20]. The experiments were conducted with a 3 mil (76.2 μm) platinum wire as the cylindrical collector and air velocities ranging from 428 to 5090 cm sec^{-1}. Monodisperse aerosols of sulfuric acid with diameters ranging from 0.56 to 1.30 μm were set up using a condensation aerosol generator. The results of these experiments are shown in Figure 10 [20], along with the theoretical curve [21] produced by the methods described in reference [17]. The conditions of the experiments were also simulated for one data point with the present calculation method, and the result plotted in Figure 10. This calculation applied the Cunningham correction factor for small particles given by equation (13). The factor for the selected particle size of 1.073 μm was applied to the particle drag coefficient in air for all calculations to obtain the result. The Cunningham correction factor was not applied to the calculated results shown in Figures 3 to 8, but the effect of neglecting it can be seen by examining Table 5. The effect of not applying the correction factor, or, in other words, considering the fluid of continuum,

is less than 2 percent for both the collection efficiency and drag coefficient for all but the smallest particle sizes considered. This certainly applies to the portions of the curves represented by solid lines for compressible flow and also for incompressible flow since the same size particles were considered.

As mentioned previously, there is also some experimental information on drag forces due to particles in air flow around cylinders [6, 7]. These experiments were performed with spherical glass beads of 0.0055, 0.0155 and 0.0470 μm diameters moving vertically toward a cylinder 0.2 cm diameter in a tube. The drag coefficients for the particles were adjusted to account for the effect of gravity in retarding the velocity of the particles [8]. The drag coefficients obtained were between 2.3 and 2.4 for ϕ between 1.5 and 15 and inertia parameters ranging from 18 to 1300, which is to be compared approximately to the upper curve in the high range of inertia parameters shown in Figure 8.

The results shown in Table 2 and plotted in Figures 5 to 8 indicate that the effect of compressibility on the drag due to particles large enough so that the air can be considered a continuum is negligible in comparison to the effect of compressibility on drag due to the air itself. This can be seen by comparison to the results shown in references [1, 2, 3, 4 and 5], which deal with drag in compressible flow. The reason that the effect of compressibility on particle trajectories and hence collection efficiency and drag coefficient is negligible is found in the comparison of the compressible air velocity field with the incompresible air velocity field. The effect of compressibility is most pronounced close to the cylinder [14]. Furthermore, the effect of compressibility is pronounced only over a small portion of the cylinder near the top and bottom, as shown in Figure 10.

CONCLUSIONS

The collection efficiences and drag coefficients of cylinders due to airborne particles have been calculated over a wide range of inertia parameters and Reynolds numbers for incompressible flow and subsonic compressible flow near critical Mach number. The results agree favourably with the small amount of experimental data available. The effect of compressibility on the drag forces due to particles carried in an air flow was found negligible for particles large enough so that the fluid can be considered a continuum.

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TABLE 1

CALCULATED COLLECTION EFFICIENCIES AND DRAG COEFFICIENTS
FOR MACH NUMBER 0.4 IN STANDARD ICAO ATMOSPHERE
AT 3048 METRES HEIGHT

INERTIA PARAMETER K	CYLINDER DIAMETER D cm	PARTICLE DIAMETER d μm	DIMENSIONLESS PARAMETER ϕ	M = 0.4		INCOMPRESSIBLE	
				E_m	C_D	E_m	C_D
0.4	1.631	0.273	1000	0.0913	0.0357	0.1040	0.0490
1	1.631	0.4317	1000	0.2621	0.2670	0.2794	0.3118
4	1.631	0.8634	1000	0.5728	1.0798	0.5896	1.1483
10	1.631	1.356	1000	0.7404	1.6536	0.7536	1.7124
40	1.631	2.730	1000	0.8924	2.2292	0.8990	2.2608
100	1.631	4.317	1000	0.9410	2.4212	0.9449	2.4401
400	1.631	8.634	1000	0.9747	2.5558	0.9763	2.5649

TABLE 2

CALCULATED COLLECTION EFFICIENCIES AND DRAG COEFFICIENTS FOR COMPRESIBLE FLOW AT MACH NUMBER 0.4 AND INCOMPRESSIBLE FLOW IN STANDARD ICAO ATMOSPHERE AT SEA LEVEL

INERTIA PARAMETER K	CYLINDER DIAMETER D cm	PARTICLE DIAMETER d µm	DIMENSIONLESS PARAMETER ϕ	M = 0.4		INCOMPRESSIBLE	
				E_m	C_D	E_m	C_D
0.4	0.02531	0.215	10	0.1384	0.0811	0.1533	0.1042
1	0.02531	0.339	10	0.3644	0.5040	0.3807	0.5588
4	0.02531	0.679	10	0.6999	1.5311	0.7110	1.5823
10	0.02531	1.073	10	0.8422	2.0471	0.8489	2.0791
40	0.02531	2.146	10	0.9454	2.4416	0.9479	2.4554
100	0.02531	3.393	10	0.9721	2.5465	0.9733	2.5545
400	0.02531	6.787	10	0.9878	2.6092	0.9882	2.6133
0.4	0.2531	0.679	100	0.1190	0.0611	0.1330	0.0802
1	0.2531	1.073	100	0.3283	0.4130	0.3451	0.4665
4	0.2531	2.146	100	0.6532	1.3589	0.6665	1.4179
10	0.2531	3.393	100	0.8056	1.9037	0.8151	1.9480
40	0.2531	6.787	100	0.9272	2.3691	0.9312	2.3889
100	0.2531	10.731	100	0.9618	2.5051	0.9639	2.5168
400	0.2531	21.462	100	0.9837	2.5924	0.9845	2.5981
0.4	2.531	2.146	1000	0.0913	0.0357	0.1040	0.0490
1	2.531	3.393	1000	0.2621	0.2671	0.2794	0.3118
4	2.531	6.787	1000	0.5729	1.0772	0.5897	1.1485
10	2.531	10.731	1000	0.7405	1.6540	0.7536	1.7127
40	2.531	21.462	1000	0.8925	2.2294	0.8991	2.2609
100	2.531	33.93	1000	0.9410	2.4213	0.9449	2.4401
400	2.531	67.87	1000	0.9747	2.5558	0.9763	2.5649
0.4	25.31	6.787	10000	0.0525	0.0122	0.0622	0.0183
1	25.31	10.731	10000	0.1779	0.1260	0.1943	0.1567
4	25.31	21.462	10000	0.4556	0.7100	0.4747	0.7823
10	25.31	33.934	10000	0.6341	1.2707	0.6513	1.3401
40	25.31	67.867	10000	0.8249	1.9596	0.8362	2.0107
100	25.31	107.308	10000	0.8970	2.2426	0.9043	2.2764
400	25.31	214.615	10000	0.9531	2.4671	0.9566	2.4847
0.4	126.57	15.176	50000	0.0306	0.0044	0.0378	0.0072
1	126.57	23.995	50000	0.1191	0.0592	0.1319	0.0759
4	126.57	47.989	50000	0.3537	0.4464	0.3733	0.5065
10	126.57	75.876	50000	0.5284	0.9099	0.5484	0.9892
40	126.57	151.756	50000	0.7441	1.6431	0.7594	1.7125
100	126.57	239.947	50000	0.8340	2.0057	0.8494	2.0551
400	126.57	479.894	50000	0.9197	2.3288	0.9262	2.3592

TABLE 3

EFFECT OF TIME STEP INCREMENT SIZE
ON COLLECTION EFFICIENCY AND
DRAG COEFFICIENT FOR VARIOUS INERTIA PARAMETERS

ICAO ATMOSPHERE AT 3048 m M = 0.4 K = 0.4 ϕ = 1000

K	η		RATIO	C_D		RATIO
	ΔT=0.1	ΔT=0.02	E_m ΔT=0.1 / E_m ΔT=0.2	ΔT=0.1	ΔT=0.02	$C_D \Delta T$=0.1 / $C_D \Delta T$=0.02
0.4	0.09127	0.06882	1.3262	0.0357	0.0210	1.7000
1	0.2621	0.2361	1.1101	0.2670	0.2194	1.2170
4	0.5728	0.5553	1.0315	1.0768	1.0158	1.0601
10	0.7404	0.7296	1.0148	1.6536	1.6077	1.0286
40	0.8924	0.8879	1.0051	2.2292	2.2069	1.0101
100	0.9410	0.9386	1.0026	2.4212	2.4087	1.0052
400	0.9742	0.9737	1.0005	2.5558	2.5507	1.0020

TABLE 4

EFFECT OF TIME STEP INCREMENT SIZE ON CALCULATED VALUES OF COLLECTION EFFICIENCY AND DRAG COEFFICIENT FOR SMALL INERTIA PARAMETER

ICAO ATMOSPHERE AT 3048 m M = 0.4 K = 0.4 ϕ = 1000

ΔT		COLLECTION EFFICIENCY		DRAG COEFFICIENT	
		CALCULATED POINT	FITTED LINE	CALCULATED POINT	FITTED LINE
0.1		0.0913	0.0912	0.0357	0.0354
0.05		0.0771	0.0773	0.0260	0.0265
0.02		0.0688	0.0690	0.0210	0.0212
0.004		0.0646	0.0645	0.0186	0.0184
0.001		0.0638	0.0637	0.0181	0.0179
FITTED LINE	r^2		0.99981		0.99750
$y = a + bx$	a		0.006340		0.01768
r^2 = coefficient of determination	b		0.2779		0.1772

TABLE 5

EFFECT OF CUNNINGHAM CORRECTION FACTOR
FOR VERY SMALL PARTICLES ON COLLECTION EFFICIENCY AND
DRAG COEFFICIENT AT MACH 0.4

PARTICLE DIAMETER d μm	CUNNINGHAM CORRECTION FACTOR C_c	INERTIA PARAMETER K	DIMENSIONLESS PARAMETER ϕ ϕ	CORRECTED / UNCORRECTED E_{mc}/E_m	CORRECTED / UNCORRECTED C_{Dc}/C_D
1.073	1.153	10	10	1.014	1.019
2.146	1.076	40	10	1.001	1.002
2.146	1.076	4	100	1.008	1.011
2.146	1.076	0.4	1000	1.106	1.275
3.393	1.048	1	1000	1.008	1.010
6.787	1.024	0.4	10000	1.002	1.008

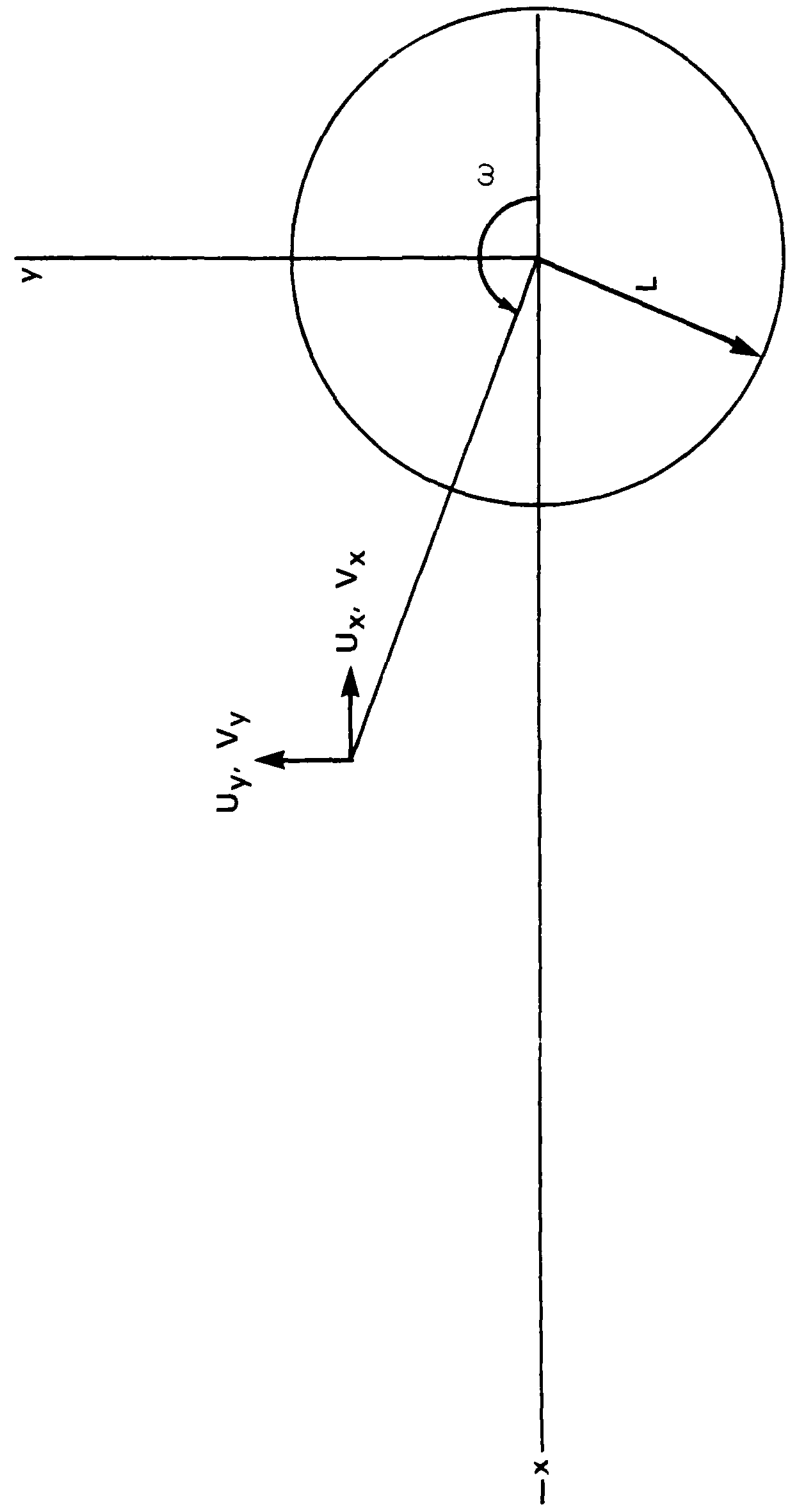

Figure 1
CO-ORDINATE SYSTEM FOR TRANVERSE FLOW WITH RESPECT TO A CIRCULAR CYLINDER

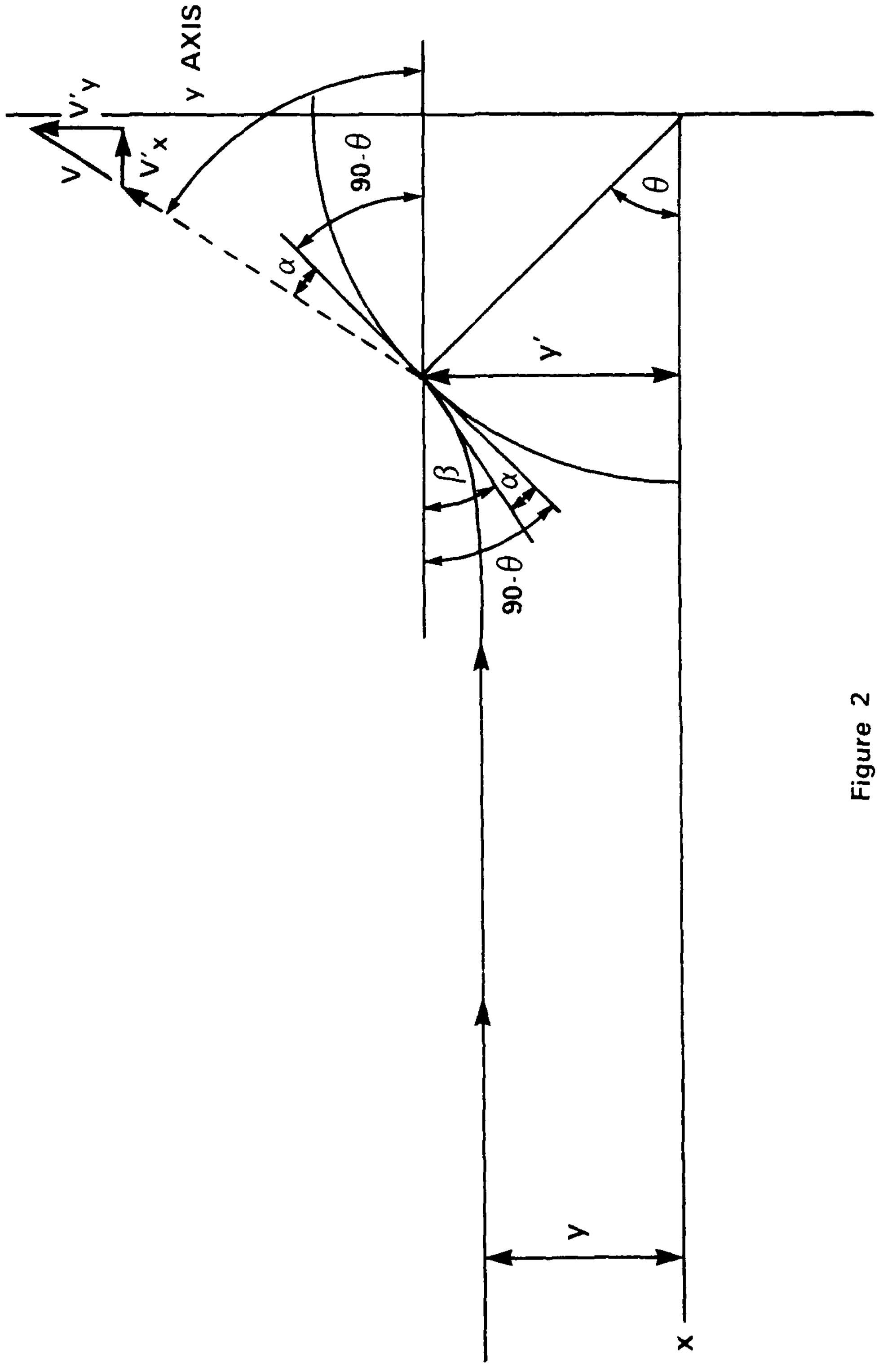

Figure 2

VELOCITY CHANGE OF A PARTICLE DUE TO INTERACTION WITH A CYLINDER

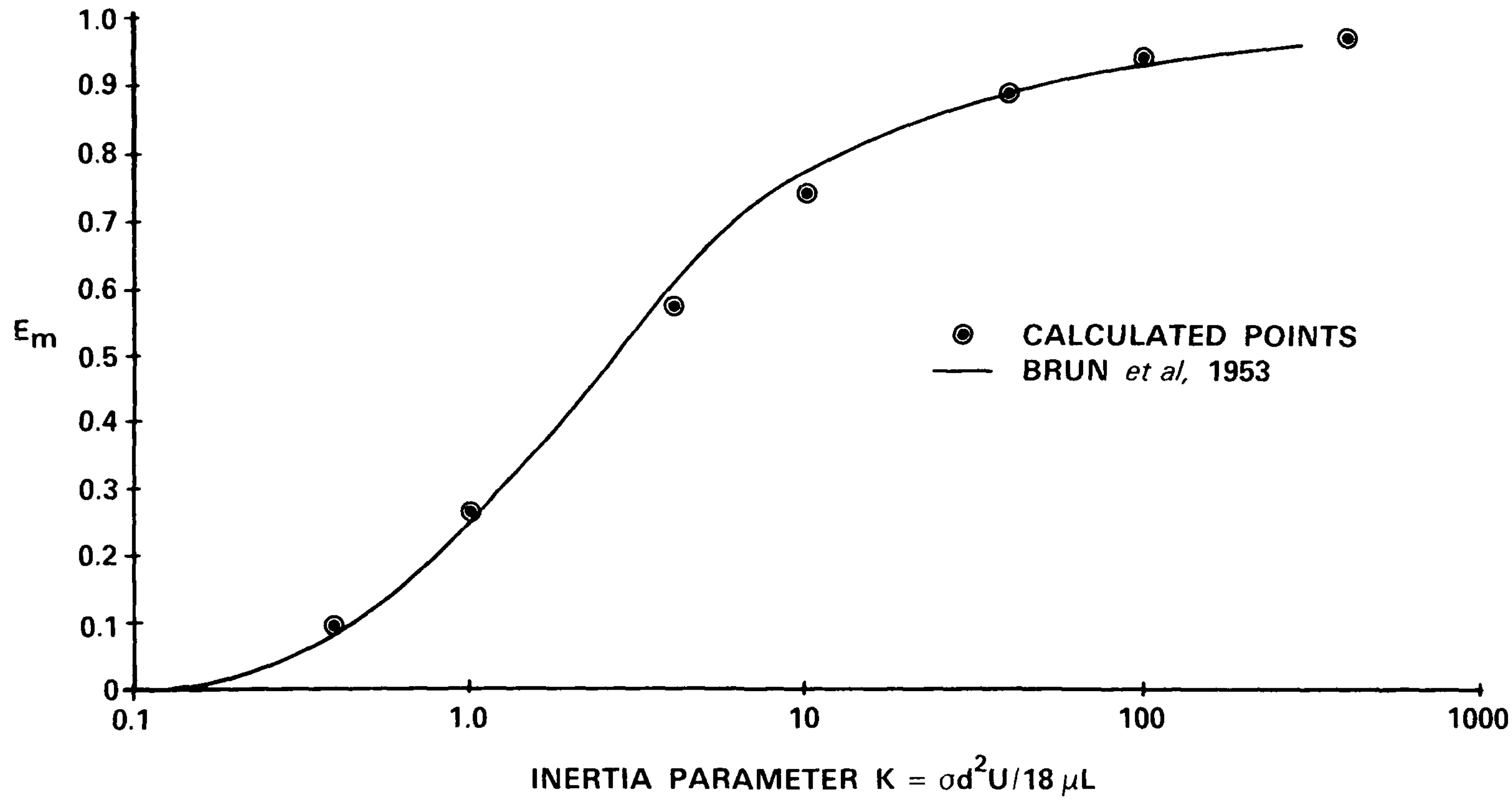

Figure 3

CYLINDER COLLECTION EFFICIENCY IN COMPRESSIBLE AIR FLOW AT MACH NUMBER OF 0.4 AND Φ OF 1000 IN ICAO STANDARD ATMOSPHERE AT 3048 METERS

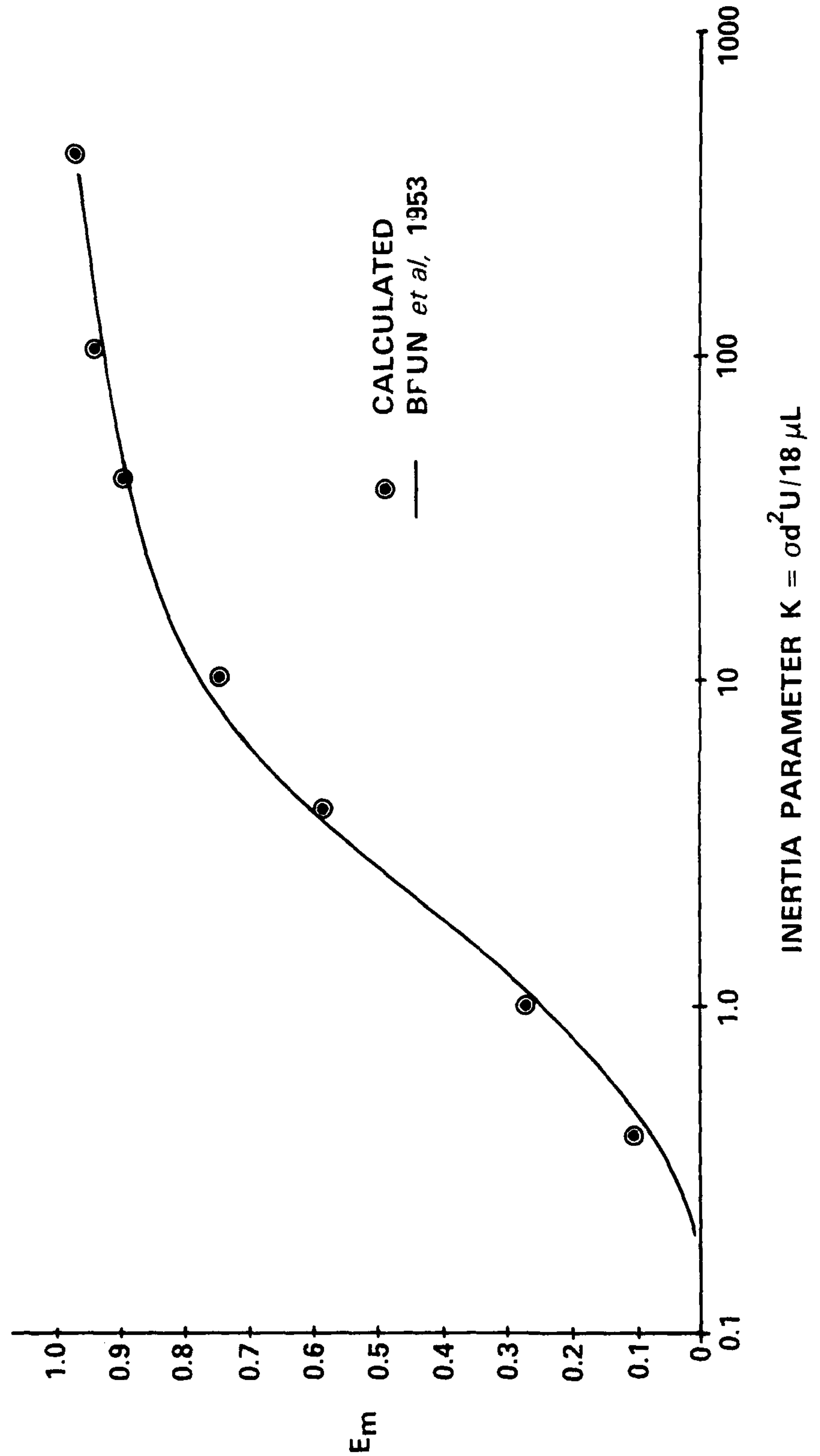

Figure 4

CYLINDER COLLECTION EFFICIENCY IN INCOMPRESSIBLE AIR FLOW AT Φ = 1000 IN ICAO STANDARD ATMOSPHERE AT 3048 METERS.

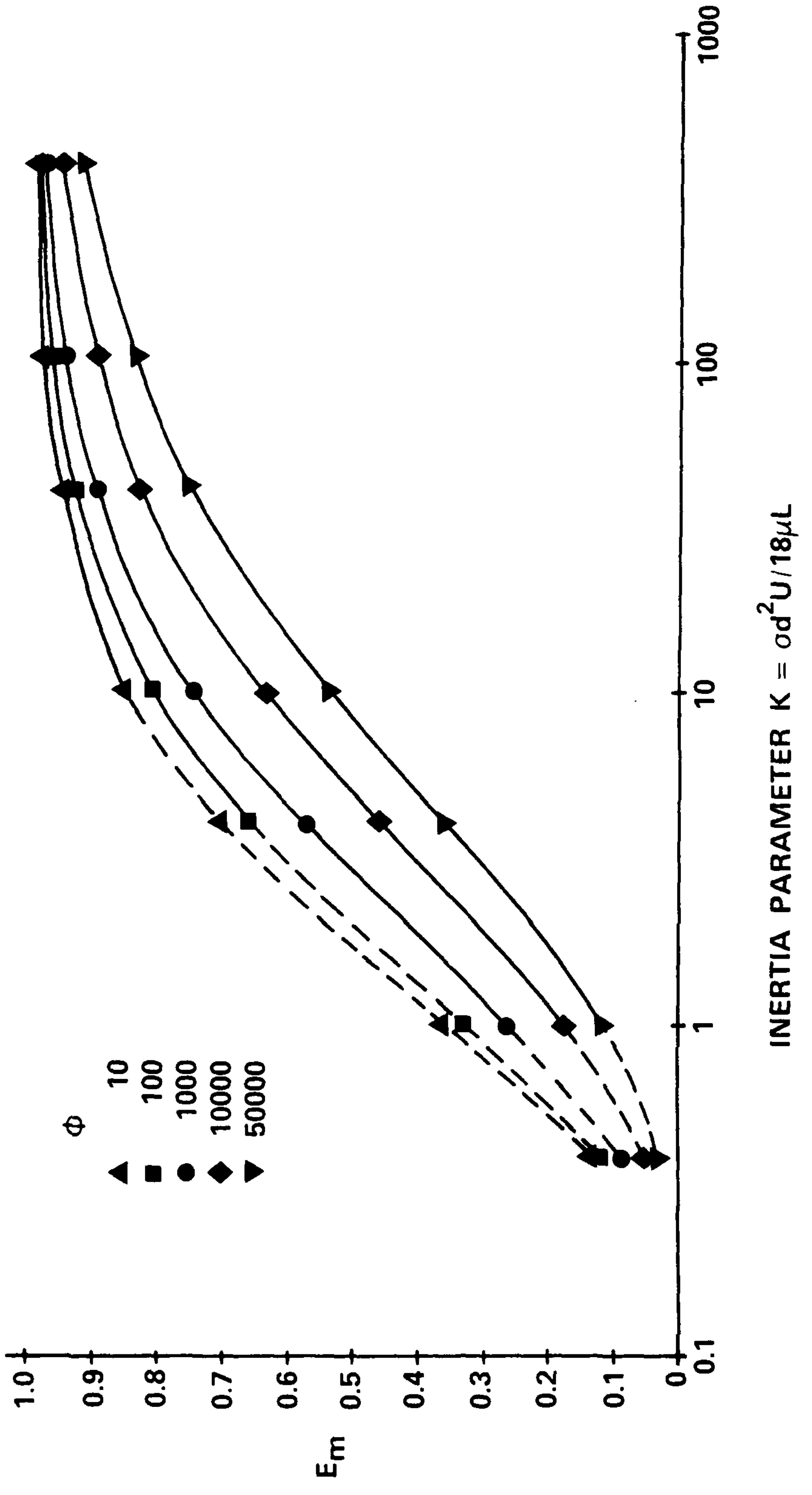

Figure 5

CYLINDER COLLECTION EFFICIENCY IN COMPRESSIBLE AIR FLOW AT MACH NUMBER OF 0.4 IN ICAO STANDARD ATMOSPHERE AT SEA LEVEL

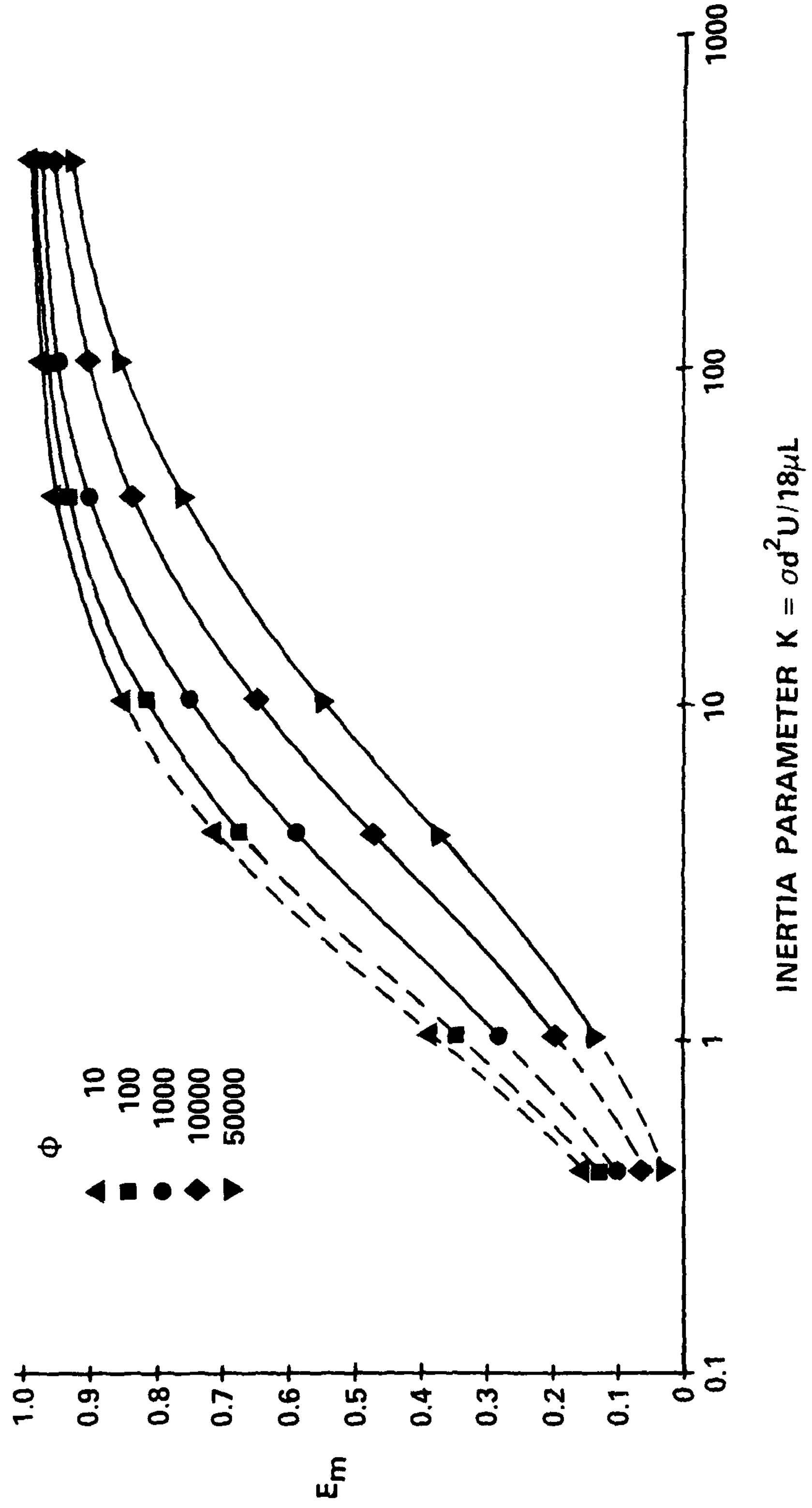

Figure 6

CYLINDER COLLECTION EFFICIENCY IN INCOMPRESSIBLE AIR FLOW IN ICAO STANDARD ATMOSPHERE AT SEA LEVEL

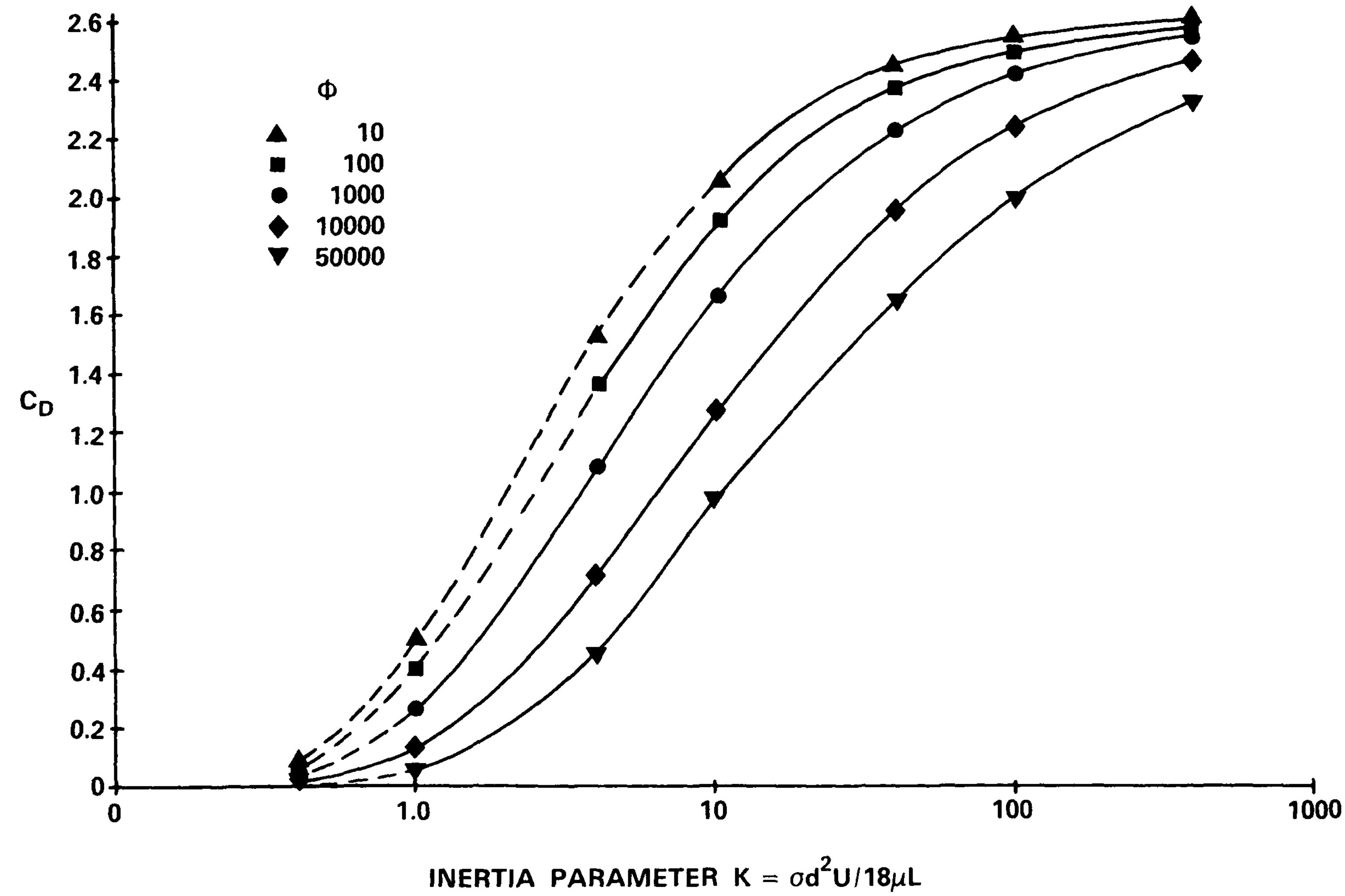

Figure 7

CALCULATED DRAG COEFFICIENT FOR CYLINDER DUE TO PARTICLES ALONE AT MACH NUMBER OF 0.4 IN ICAO STANDARD ATMOSPHERE AT SEA LEVEL

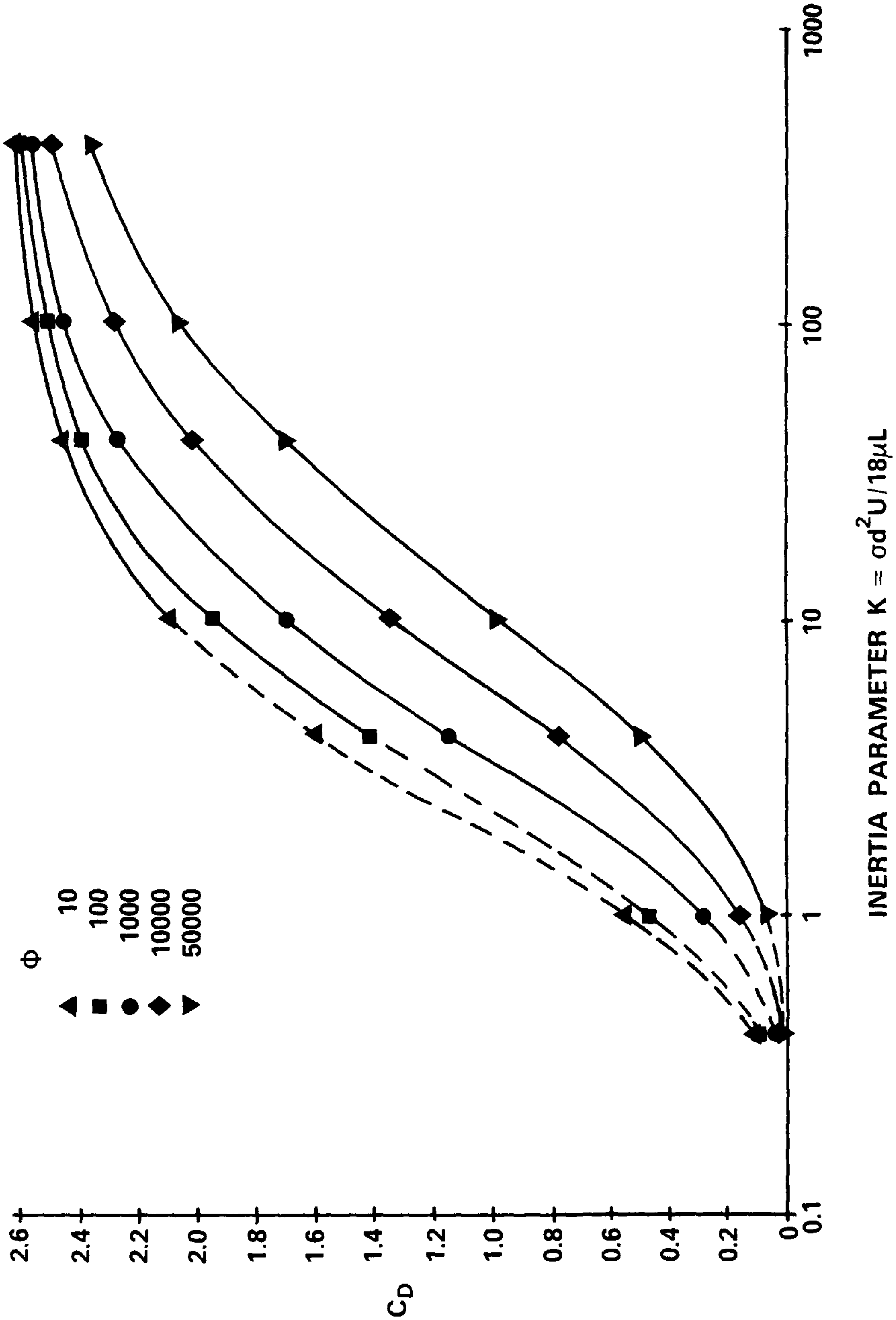

Figure 8

CALCULATED DRAG COEFFICIENT FOR CYLINDERS DUE TO PARTICLES ALONE IN INCOMPRESSIBLE FLOW IN ICAO STANDARD ATMOSPHERE AT SEA LEVEL

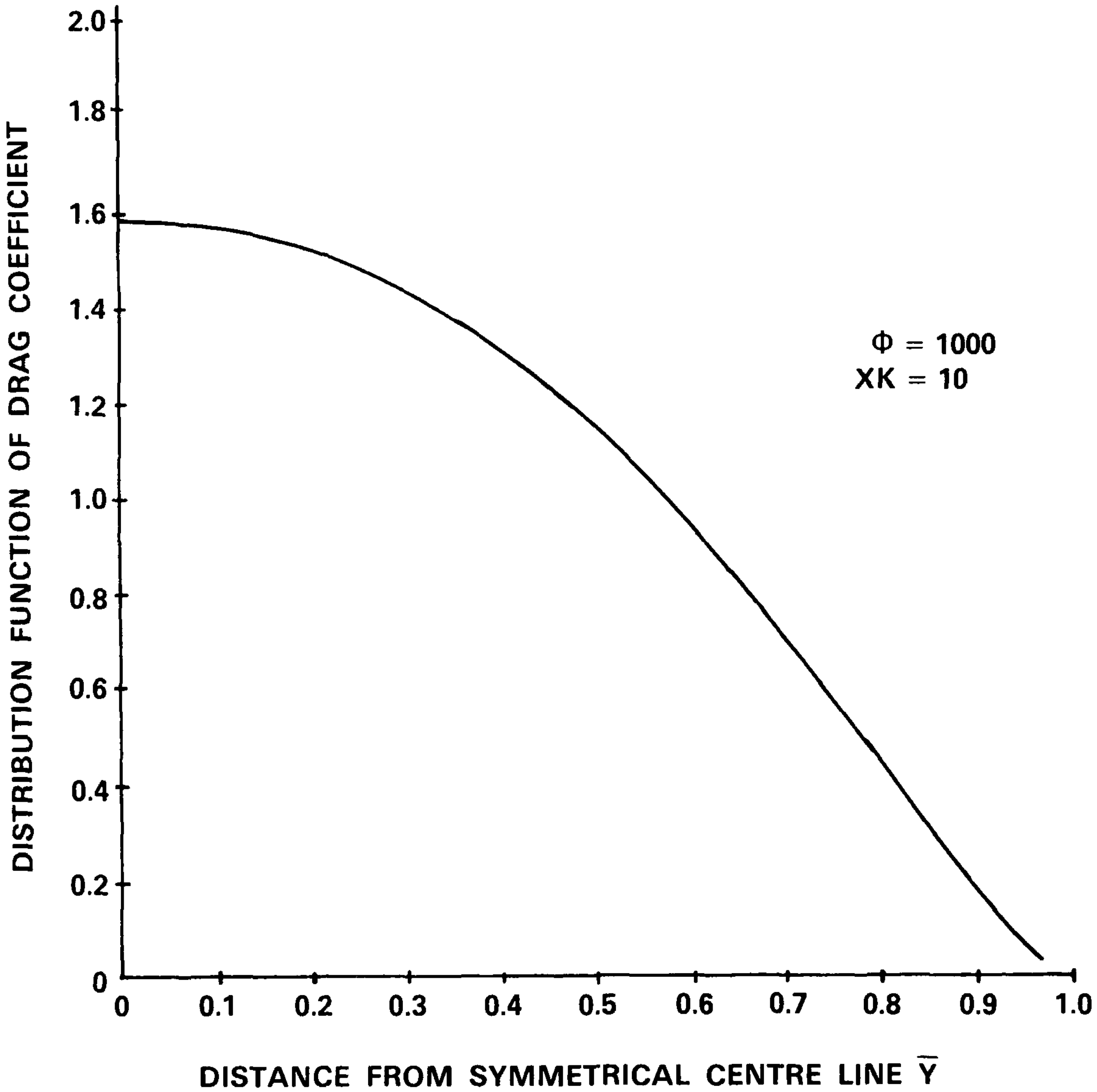

Figure 9

DISTRIBUTION OF FORCE DUE TO ELASTIC REFLECTION OF PARTICLES FROM A CYLINDER AT MACH NO. OF 0.4 in ICAO STANDARD ATMOSHERE AT SEA LEVEL

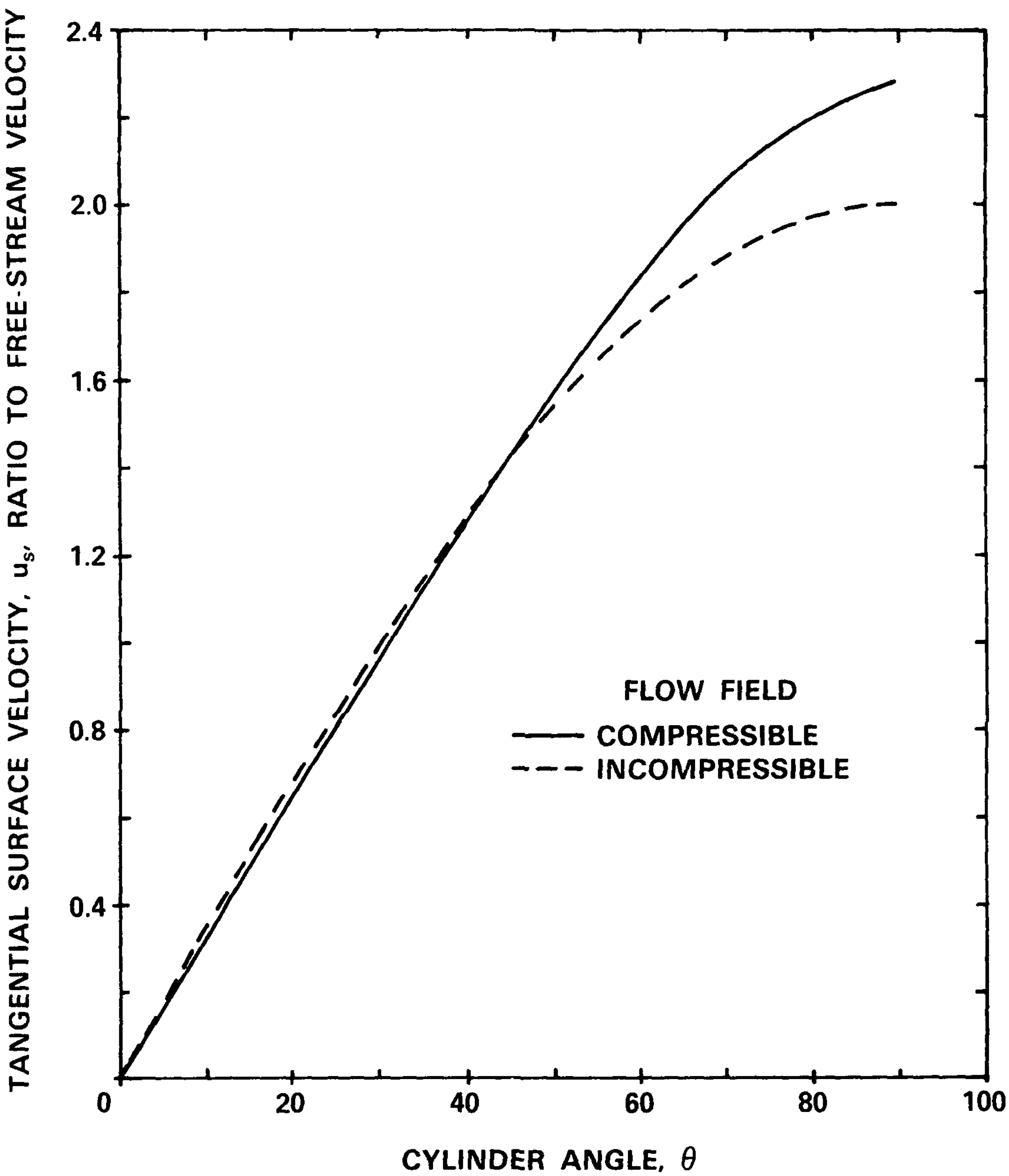

Figure 10

COMPARISON OF SURFACE VELOCITY ON A CYLINDER BETWEEN INCOMPRESSIBLE FLOW FIELD AND COMPRESSIBLE-AIR FLOW FIELD AT MACH NO. 0.4

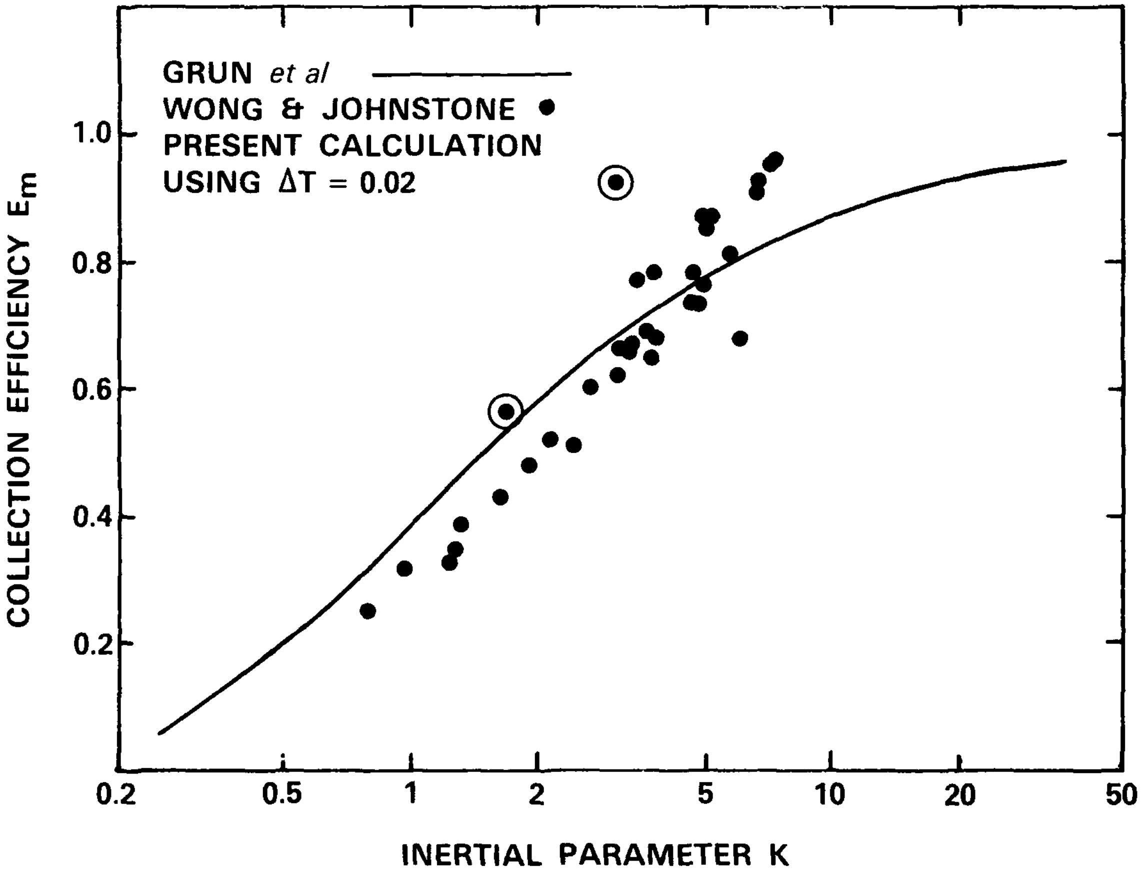

Figure 11

COMPARISON OF PRESENT CALCULATION METHOD TO THEORY AND EXPERIMENT OF OTHER INVESTIGATORS

```
08:49 NOV 01 '88 IMPACTCC.CHM00100

      PROGRAM IMPACTCC
C
C     ESTABLISH PHYSICAL PROPERTIES FOR CALCULATING IMPACTION EFFICIENCY
C
C     ESTABLISH PARAMETERS FOR STEP BY STEP INTEGRATION AND HALF
C     INTERVAL METHOD
C     G4LFT AND G4RIT ARE LOWER AND UPPER BOUNDS RESPECTIVELY FOR
C     STARTING LOCATION OF PARTICLE
C     SIGNL IS SIGN CORRECTION FOR HALF INTERVAL METHOD
C     IN THIS PROBLEM IT IS -1.0
C     DTAU  IS THE TIME INCREMENT FOR STEP BY STEP INTEGRATION
C     NIBP IS INTERVAL OF WRITTEN PARTICLE PATHS
C     NSBP IS INTERVAL OF WRITTEN INTEGRATION STEPS
C     NX IS NUMBER OF ITERATIONS USED TO DETERMINE PARTICLE STARTING
C     POINT


    1 READ(2,100)G4LFT,G4RIT,SIGNL,DTAU,NIBP,NSBP,NX
C
C     ESTABLISH PHYSICAL PROPERTIES FOR CALCULATING IMPACTION EFFICIENCY
C
C     DC    IS CYLINDER DIAMETER, CM
C     DP    IS PARTICLE DIAMETER, CM
C     SIGMA IS PARTICLE DENSITY, G/CC
C     XMU   IS ABSOLUTE VISCOSITY OF FLUID, POISE
C     UF    IS FREE STEAM VELOCITY, CM/SEC
C     FSP IS FREE STREAM PRESSURE, MILLIBARS
C     FST IS FREE STREAM TEMPERATURE, K
C
C     RHO IS FLUID DENSITY G/CC
C
      READ(2,101)DC,DP,SIGMA,XMU,UF
      READ(2,105) FSP,FST
C
C     XL IS UPSTREAM STARTING POSITION FOR CALCULATING PARTICLE MOTION
C
      READ(2,102)XL
C
      RHO = FSP/(2871.3*FST)
      REZ=RHO*DP*UF/XMU
      XK=SIGMA*DP**2*UF/(9.*XMU*DC)
      P = 9.*RHO**2*UF*DC/(XMU*SIGMA)
      XMACH = UF/(2005.0*SQRT(FST))
C
      WRITE(3,200)
      WRITE(3,201)
```

```
C
C     XK IS INERTIAL PARAMETER CALLED STOKES NUMBER
C     REZ IS FREE STREAM REYNOLDS NUMBER OF SPHERICAL PARTICLE
C     P IS DIMENSIONLESS GROUP INDEPENDENT OF DROP SIZE
C     XMACH IS FREE STREAM MACH NUMBER
C
      WRITE(3,202)XL,REZ,XK,P,DC,DP,RHO,SIGMA,XMU,UF,FSP,FST,XMACH
C
C     CALCULATE STARTING POSITION OF TANGENT PARTICLE
C
      CALL SBM29(G4LFT,G4RIT,SIGNL,DTAU,NIBP,NSBP,NX,XL,XK,REZ,G4ZER,
     1XMACH)
C
C     ESTABLISH NUMBER OF INCREMENTS FOR INTEGRATING PARTICLE FORCE
C
C     NZER IS NUMBER OF INCREMENTS
C     NJBP IS INTERVAL OF WRITTEN INCREMENTS
C
      READ(2,103)NZER,NJBP
C
C     CALCULATE FORCE PRODUCED BY PARTICLES
C
      CALL SBM26(XL,G4ZER,DTAU,XK,REZ,PX,NZER,NJBP,XMACH)
C
C
C     RECYL IS REYNOLDS NUMBER OF CYLINDER IN FREE AIR
C
      RECYL=RHO*DC*UF/XMU
      WRITE(3,204)RECYL
      READ(2,111)NSTOP
      IF(NSTOP)1,30,30
   30 CONTINUE
C
C     FORMATS FOR INPUT AND OUTPUT STATEMENTS
C
  100 FORMAT(10X,F10.7,20X,F10.7,19X,F3.0/ 10X,F10.7,20X,I3,27X,I2/
     1   10X,I2)
  101 FORMAT(F10.5,F10.7,F10.6, F10.7,F10.1)
  102 FORMAT (F10.7)
  103 FORMAT(I5,I5)
  105 FORMAT(F10.2,F10.2)
  111 FORMAT(I5)
  200 FORMAT( 1H1, 34X, 51HTHE DRAG ON CYLINDERS IN A STREAM OF DUST-LAD
     1EN AIR / 1H0 )
  201 FORMAT(  28H0THE PHYSICAL PARAMETERS ARE )
  202 FORMAT(  10H0XL      =  ,F10.6/10H REZ     =  ,E12.6/10H XK      =  ,
     1   E12.6/10H P       =  ,E12.6/
```

```
     2   10HODC     =  , F10.5/10H DP    =  ,F10.7/10H RHO    =  ,F10.6/
     3   10H SIGMA =  , F10.6/10H XMU   =  ,F10.7/10H UF     =  ,F10.1/
     4   10H FSP    =  , F10.2/10H FST   =  ,F10.2/10H XMACH =  ,F10.6)
  204 FORMAT(  32HOTHE TARGET REYNOLDS NUMBER IS  ,E10.4)
      END
      SUBROUTINE  SBM38(G,XMACH,UX,UY)
C
C     THIS SUBROUTINE CALCULATES THE COMPRESSIBLE AIR
C     FLOW AROUND A CIRCULAR CYLINDER.
C
      DIMENSION G(4)
      R = SQRT(G(3)**2 + G(4)**2)
      THETA = ASIN(G(4)/R)
      PI = 3.14159265358979323846264
      A = PI - THETA
C
C     CALCULATE VELOCITY COMPONENTS INDEPENDENT OF MACH NUMBER
C
      UX1=(1.-1./R**2)*COS(A)**2+(1.+1./R**2)*SIN(A)**2
      UY1=(-1./R**2)*SIN(2.*A)
C
C     CALCULATE RADIAL COMPONENTS OF COEFFICENTS OF M**2
C
      FR1=(-13./12.)/R**2+1.5/R**4-(5./12.)/R**6
      FR2=.25/R**2-.25/R**4
      FR3=(-13./12.)/R**2+.5/R**4-(1./12.)/R**6
      FR4=.75/R**2-.25/R**4
C
C     CALCULATE RADIAL COMPONENTS OF COEFFICIENTS OF M**4
C
      FR5=(-17./60.)/R**2+(3./8.)/R**4-(5./12.)/R**6+(7./16.)/R**8
     1-(9./80.)/R**10
      FR6=(61./80.)/R**4-(15./16.)/R**6+(7./40.)/R**8
      FR7=(-3./16.)/R**4+(3./16.)/R**6
      FR8=(-137./80.)/R**2+4./R**4-(65./16.)/R**6+(35/16.)/R**8
     1-(33./80.)/R**10
      FR9=(19./48.)/R**2+(5./16.)/R**4-(15./16.)/R**6
     1+(7./24.)/R**8-(1./16.)/R**10
      FR10=(-1./16.)/R**2-(3./16.)/R**4+(1./4.)/R**6
      FR11=(-17./60.)/R**2+(1./8.)/R**4-(1./12.)/R**6+
     1(1./16.)/R**8-(1./80.)/R**10
      FR12=(61./80.)/R**4-(9./16.)/R**6+(3./40.)/R**8
      FR13=(-5./16.)/R**4+(3./16.)/R**6
      FR14=(-137./80.)/R**2+(4./3.)/R**4-(13./16.)/R**6
     1+(5./16.)/R**8-(11./240.)/R**10
      FR15=(19./16.)/R**2+(5./16.)/R**4-(9./16.)/R**6
     1+(1./8.)/R**8-(1./48.)/R**10
      FR16=(-5./16.)/R**2-(5./16.)/R**4+(1./4.)/R**6
```

```
C
C     CALCULATE COMPLETE VELOCITY COMPONENTS
C
      UX2=FR1*COS(A)**2+FR2*COS(A)*COS(3.*A)-FR3*SIN(A)**2
     1-FR4*SIN(A)*SIN(3.*A)
C
      UX3=0.4*(FR5*COS(A)**2+FR6*COS(A)*COS(3.*A)+FR7*COS(A)*COS(5.*A)
     2)+FR8*COS(A)**2+FR9*COS(A)*COS(3.*A)+FR10*COS(A)*COS(5.*A)
     3-0.4*(FR11*SIN(A)**2+FR12*SIN(A)*SIN(3.*A)+FR13*SIN(A)*SIN(5.*A)
     3)-FR14*SIN(A)**2-FR15*SIN(A)*SIN(3.*A)-FR16*SIN(A)*SIN(5.*A)
C
      UX=UX1+UX2*XMACH**2+UX3*XMACH**4
C
      UY2=FR1*SIN(A)*COS(A)+FR2*SIN(A)*COS(3.*A)+
     1FR3*SIN(A)*COS(A)+FR4*COS(A)*SIN(3.*A)
C
      UY3=.4*(FR5*SIN(A)*COS(A)+FR6*SIN(A)*COS(3.*A)+FR7*SIN(A)*COS(5.
     1*A))+FR8*SIN(A)*COS(A)  +  FR9*SIN(A)*COS(3.*A)
     2+FR10*SIN(A)*COS(5.*A)+0.4*(FR11*SIN(A)*COS(A)+FR12*SIN(3.*A)*C
     3OS(A)+FR13*SIN(5.*A)*COS(A))+FR14*SIN(A)*COS(A)+FR15*SIN(3.*A)*
     4COS(A)+FR16*SIN(5.*A)*COS(A)
C
      UY=UY1+UY2*XMACH**2+UY3*XMACH**4
C
      RETURN
      END
C
      SUBROUTINE SBM28(G,TAU,DTAW,XK,REZ,UX,UY,XCDRE,XMACH)
C
C     THIS SUBROUTINE CALCULATES PARTICLE MOTION DURING THE FINAL
C     INCREMENT OF TAU
C
      DIMENSION G(4),DG(4)
      M=0
C
C     CALL ON RUNGE KUTTA SUBROUTINE
C
    8 CONTINUE
      M=M+1
      CALL SBM22(4,G,DG,TAU,DTAW,IRUNG,M)
      IF(IRUNG-1)10,9,10
    9 RHORAT=((1.+.2*XMACH**2)/(1.+.2*(UX**2+UY**2)*XMACH**2))**2.5
      REZC = REZ*RHORAT
      RE=REZC*((UY-G(2))**2+(UX-G(1))**2)**0.5
      XCDRE=CDRE(RE)
      DG(1)=(XCDRE/(24.0*XK))*(UX-G(1))
      DG(2)=(XCDRE/(24.0*XK))*(UY-G(2))
      DG(3)=G(1)
```

```
      DG(4)=G(2)
      GO TO 8
   10 CONTINUE
      M=0
C
C     CALCULATE FLUID VELOCITY AT PARTICLE POSITION
C
      CALL SBM38(G,XMACH,UX,UY)
C
C     INTEGRATE ACROSS ANOTHER STEP IF REQUIRED
C
      IF(G(4) - 1.0)11,11,12
   11 DELX = SQRT(1.0 - G(4)**2)
      GO TO 13
   12 DELX = 0.0
   13 HITS=G(3)+G(1)*1.1*DTAW+DELX
      IF(HITS)8,8,18
   18 CONTINUE
      RETURN
      END
C
      FUNCTION CDRE(RE)
C
C     THIS FUNCTION COMPUTES THE PRODUCT OF DRAG COEFFICIENT
C     AND REYNOLDS NUMBER FOR A SPHERE AS A FUNCTION OF
C     REYNOLDS NUMBER
C
C     CONSTANT COEFFICIENTS
C
      A1=1./24.
      A2=-2.3363*1.E-04
      A3=2.0154*1.E-06
      A4=-6.9105*1.E-09
      B0=-1.29536
      B1=9.86*1.E-01
      B2=-4.6677*1.E-02
      B3=1.1235*1.E-03
C
C     CHOOSE THE APPROPRIATE POLYNOMIAL
C
      IF(RE-4.0)2,7,7
C
C     INITIAL ESTIMATE
C
    2 IF(RE - 0.001)3,4,4
    3 CDRE = 24.0
C     CDRE = CDRE / 1.153
      GO TO 30
```

```
    4 X=24.*RE
C
C     BEGIN NEWTON METHOD ITERATION
C
      CONTINUE
      DO 6 ITER=1,20
      FX=A1*X+A2*X**2+A3*X**3+A4*X**4-RE
      FPX=A1+2.*A2*X+3.*A3*X**2+4.*A4*X**3
      DELX=FX/FPX
      X=X-DELX
C
C     CHECK FOR CONVERGENCE
C
      EPS=1.E-06
      IF(ABS(DELX/X)-EPS)5,5,6
    5 CDRE=X/RE
C     CDRE = CDRE / 1.153
      GO TO 30
    6 CONTINUE
      GO TO 29
C
C     INITIAL ESTIMATE
C
    7 CD = 1.0
      ELOG = 0.434294481903252
      X=ALOG(CD*RE**2)*ELOG
C
C     BEGIN NEWTON METHOD ITERATION
C
      DO 24 ITER=1,20
      FX=B0+B1*X+B2*X**2+B3*X**3 - ALOG(RE)*ELOG
      FPX=B1+2.*B2*X+3.*B3*X**2
      DELX=FX/FPX
      X=X-DELX
C
C     CHECK FOR CONVERGENCE
C
      EPS=1.E-06
      IF(ABS(DELX/X)-EPS)22,22,24
   22 CDRE=10.**X/RE
      GO TO 30
   24 CONTINUE
   29 WRITE(3,202)
   30 RETURN
C
C     FORMATS FOR OUTPUT STATEMENTS
C
  202 FORMAT(16H0 NO CONVERGENCE)
```

```
C
      END
      SUBROUTINE SBM22(N,Y,F,X,H,IRUNG,M)
C                                                                       SBM22001
C     FOURTH ORDER RUNGE KUTTA METHOD                                   SBM22002
C     FOR N FIRST ORDER O.D.E.                                          SBM22003
C                                                                       SBM22005
      DIMENSION PHI(50),SAVY(50),Y(50),F(50)                            SBM22004
      GO TO (2,3,4,5,6),M                                               SBM22006
C                                                                       SBM22007
C     PASS 1                                                            SBM22008
C                                                                       SBM22009
    2 IRUNG=1                                                           SBM22010
      RETURN                                                            SBM22011
C                                                                       SBM22012
C     PASS 2                                                            SBM22013
C                                                                       SBM22014
    3 DO 22 J=1,N                                                       SBM22015
      SAVY(J)=Y(J)
      PHI(J)=F(J)                                                       SBM22017
   22 Y(J)=SAVY(J)+0.5*H*F(J)                                           SBM22018
      X=X+0.5*H                                                         SBM22019
      IRUNG=1                                                           SBM22020
      RETURN                                                            SBM22021
C     PASS 3                                                            SBM22022
    4 DO 33 J=1,N                                                       SBM22023
      PHI(J)=PHI(J)+2.0*F(J)                                            SBM22024
   33 Y(J)=SAVY(J)+0.5*H*F(J)                                           SBM22025
      IRUNG=1                                                           SBM22026
      RETURN                                                            SBM22027
C                                                                       SBM22028
C     PASS 4                                                            SBM22029
    5 DO 44 J=1,N                                                       SBM22030
      PHI(J)=PHI(J)+2.0*F(J)                                            SBM22031
   44 Y(J)=SAVY(J)+H*F(J)                                               SBM22032
      X=X+0.5*H                                                         SBM22033
      IRUNG=1                                                           SBM22034
      RETURN                                                            SBM22035
C     PASS 5                                                            SBM22036
    6 DO 55 J=1,N                                                       SBM22037
   55 Y(J) = SAVY(J) + (PHI(J) + F(J))*H/6.0
      IRUNG=2                                                           SBM22040
      RETURN                                                            SBM22041
C                                                                       SBM22042
      END                                                               SBM22043
      SUBROUTINE SBM29(G4LFT,G4RIT,SIGNL,DTAU,NIBP,NSBP,NX,XL,XK,REZ,
     1G4ZER,XMACH)
C
```

```
C     THIS SUBROUTINE CALCULATES THE
C     IMPACTION EFFICIENCY OF A CIRCULAR CYLINDER
C
      DIMENSION G(4),DG(4)
      WRITE(3,200)
      WRITE(3,201)G4LFT,G4RIT,SIGNL,DTAU,NIBP,NSBP,NX
C
C     HALF INTERVAL ITERATION FOR INITIAL G4 VALUE
C
      DO 47 ITER=1,NX
C
C     SET AND PRINT INITIAL CONDITIONS
C
      M=0
      NSTEP=0
      TAU=0.0
      G(3)=XL
      G4ZER=(G4LFT+G4RIT)/2.0
      G(4)=G4ZER
      CALL SBM38(G,XMACH,UX,UY)
      G(1)=UX
      G(2)=UY
      RHORAT=((1.+.2*XMACH**2)/(1.+.2*(UX**2+UY**2)*XMACH**2))**2.5
      REZC = REZ*RHORAT
      RE=REZC*((UY-G(2))**2+(UX-G(1))**2)**0.5
      XCDRE=CDRE(RE)
      IP=ITER/NIBP*NIBP
      IF(IP-ITER)5,7,5
    5 CONTINUE
      IF(ITER-1)6,7,6
    6 CONTINUE
      IF(ITER-NX)8,7,8
    7 CONTINUE
      WRITE (3,205)
      WRITE(3,203)ITER,G4LFT,G4ZER,G4RIT,TAU,G(1),G(2),G(3),G(4),UX,UY,
     1XCDRE
C
C     CALL ON RUNGE KUTTA SUBROUTINE
C
    8 CONTINUE
      M=M+1
      CALL SDM22(4,G,DG,TAU,DTAU,IRUNG,M)
      IF(IRUNG-1)10,9,10
    9 RHORAT=((1.+.2*XMACH**2)/(1.+.2*(UX**2+UY**2)*XMACH**2))**2.5
      REZC = REZ*RHORAT
      RE=REZC*((UY-G(2))**2+(UX-G(1))**2)**0.5
      XCDRE=CDRE(RE)
      DG(1)=(XCDRE/(24.0*XK))*(UX-G(1))
```

```
      DG(2)=(XCDRE/(24.0*XK))*(UY-G(2))
      DG(3)=G(1)
      DG(4)=G(2)
      GO TO 6
   10 CONTINUE
      M=0
C
C     CALCULATE FLUID VELOCITY AT PARTICLE POSITION
C
      CALL SBM38(G,XMACH,UX,UY)
C
C     PRINT SOLUTIONS
C
      IS = ITER/NIBP*NIBP
      IF(IS-ITER)11,13,11
   11 CONTINUE
      IF(ITER-1)12,13,12
   12 CONTINUE
      IF(ITER-NX)16,13,16
   13 CONTINUE
      NSTEP=NSTEP+1
      IF(NSTEP-NSBP)16,14,16
   14 CONTINUE
      NSTEP=0
      TAW = TAU+0.0001
      WRITE(3,204)TAW,G(1),G(2),G(3),G(4),UX,UY,XCDRE
C
C     INTEGRATE ACROSS ANOTHER STEP IF REQUIRED
C
   16 CONTINUE
      IF(G(4)-1.0)17,17,18
   17 DELX=SQRT(1.0-G(4)**2)
      GO TO 19
   18 DELX=0.0
   19 HITS=G(3)+G(1)*1.1*DTAU+DELX
      IF(HITS)8,8,20
   20 CONTINUE
C
C     CHANGE INCREMENT SIZE NEAR CYLINDER AND INTEGRATE FURTHER
C
      DTAW=DTAU/10.0
      CALL SBM28(G,TAU,DTAW,XK,REZ,UX,UY,XCDRE,XMACH)
C
C     PRINT SOLUTIONS
C
      IF(IS-ITER)21,23,21
   21 CONTINUE
      IF(ITER-1)22,23,22
```

```
   22 CONTINUE
      IF(ITER-NX)24,23,24
   23 CONTINUE
      WRITE(3,204)TAU,G(1),G(2),G(3),G(4),UX,UY,XCDRE
   24 CONTINUE
      DTAW=DTAU/100.0
      CALL SBM28(G,TAU,DTAW,XK,REZ,UX,UY,XCDRE,XMACH)
C
C     PRINT SOLUTIONS
C
      IF(IS-ITER)31,33,31
   31 CONTINUE
      IF(ITER-1)32,33,32
   32 CONTINUE
      IF(ITER-NX)34,33,34
   33 CONTINUE
      WRITE(3,204)TAU,G(1),G(2),G(3),G(4),UX,UY,XCDRE
   34 CONTINUE
C
C     CALCULATE ORDINATE AT TANGENT POINT OF TANGENT PATH
C
      ORD = G(1)/SQRT(G(1)**2 + G(2)**2)
C
C     FIND INTERVAL HALF WITH THE SIGN CHANGE
C
      IF((G(4)-ORD)*SIGNL-0.0)45,45,46
   45 G4RIT=G4ZER
      GO TO 47
   46 G4LFT=G4ZER
   47 CONTINUE
      EM = G4ZER
      WRITE(3,207) EM
C
      RETURN
C
C     FORMATS FOR OUTPUT STATEMENTS
C
  200 FORMAT( 1H0, 35X, 44H IMPACTION EFFICIENCY OF A CIRCULAR CYLINDER/
     1   1H0 )
  201 FORMAT(  10H0G4LEF =  ,F10.6/ 10H G4RIT =  ,F10.6/10H SIGNL =  ,
     1   F3.0/ 10H DTAU  =  ,F10.6/ 10H NIBP  =  ,I3/ 10H NSBP  =  ,I3/
     2   10H NX    =  ,I3)
  203 FORMAT( 10H0ITER  =  ,I3/ 10H G4LEF =  ,F10.6/ 10H G4ZER =  ,
     1   F10.6/ 10H G4RIT =  ,F10.6/ 7H0   TAU, 11X, 4HG(1), 12X,
     2   4HG(2), 12X, 4HG(3), 12X, 4HG(4), 14X, 2HUX, 14X,  2HUY ,
     3   12X, 4HCDRE /
     4   1H0, F7.4, 4F16.6, 3F16.4 )
  204 FORMAT ( 1H , F7.4, 4F16.6, 3F16.4 )
```

```
  205 FORMAT ( 46H1THE MOTION OF A CRITICAL PARTICLE IS GIVEN BY )
  207 FORMAT( 30H0THE IMPACTION EFFICIENCY IS  ,E10.4)
      END
      SUBROUTINE SBM26(XL,G4ZER,DTAU,XK,REZ,PX,NZER,NJBP,XMACH)
C
C     THIS SUBROUTINE CALCULATES THE MOTION OF PARTICLES
C     IN A FLUID STREAM MOVING TOWARD A CIRCULAR CYLINDER AND
C     CALCULATES THE FORCE OF PARTICLE IMPACT ON THE CYLINDER
C
      DIMENSION G(4),DG(4)
      DIMENSION YZER(500)
C
C     SET NUMBER OF INCREMENTS AT INITIAL POSITION
C
      WRITE(3,200)
      WRITE(3,201)NZER,NJBP
      WRITE(3,202)
      NCE=NZER+1
      DELY=G4ZER/FLOAT(NZER)
      FSUM=0.0
C
C     STEPWISE INTEGRATION FOLLOWING PARTICLE POSITION
C
      DO 40 ITER=1,NCE
C
C     SET AND PRINT INITIAL CONDITIONS
C
      M=0
      TAU=0.0
      G(3)=XL
      YZER(ITER)=FLOAT(ITER-1)*DELY
      G4ZER=YZER(ITER)
      G(4)=G4ZER
      CALL SBM38(G,XMACH,UX,UY)
      UXZER=UX
      G(1)=UX
      G(2)=UY
C
C     CALL ON RUNGE KUTTA SUBROUTINE
C
    8 CONTINUE
      M=M+1
      CALL SBM22(4,G,DG,TAU,DTAU,IRUNG,M)
      IF(IRUNG-1)10,9,10
    9 RHORAT=((1.+.2*XMACH**2)/(1. +.2*(UX**2+UY**2)*XMACH**2))**2.5
      REZC = REZ*RHORAT
      RE=REZC*((UY-G(2))**2+(UX-G(1))**2)**0.5
      XCDRE=CDRE(RE)
```

```
      DG(1)=(XCDRE/(24.0*XK))*(UX-G(1))
      DG(2)=(XCDRE/(24.0*XK))*(UY-G(2))
      DG(3)=G(1)
      DG(4)=G(2)
      GO TO 8
   10 CONTINUE
      M=0
C
C     CALCULATE FLUID VELOCITY AT PARTICLE POSITION
C
      CALL SBM38(G,XMACH,UX,UY)
C
C     INTEGRATE ACROSS ANOTHER STEP IF REQUIRED
C
      IF(G(4) - 1.0)11,11,12
   11 DELX = SQRT(1.0 - G(4)**2)
      GO TO 13
   12 DELX = 0.0
   13 HITS=G(3)+G(1)*1.1*DTAU+DELX
      IF(HITS)8,8,18
   18 CONTINUE
C
C     CHANGE INCREMENT SIZE NEAR CYLINDER AND INTEGRATE FURTHER
C
      DTAW=DTAU/10.0
      CALL SBM28(G,TAU,DTAW,XK,REZ,UX,UY,XCDRE,XMACH)
      DTAW=DTAU/100.0
      CALL SBM28(G,TAU,DTAW,XK,REZ,UX,UY,XCDRE,XMACH)
C
C     CO-ORDINATES, VELOCITY, AND PRESSURE DERIVATIVE AT CYLINDER
C
      VX=G(1)
      VY=G(2)
      X=G(3)
      Y=G(4)
      V=SQRT(VX**2+VY**2)
      PI = 3.14159265358979323846264
      GAMMA = PI - 2.0*ATAN(Y/SQRT(1.0-Y**2))- ATAN(VY/VX)
      FY= UXZER*(VX- V*COS(GAMMA))
      IF(ITER-1)24,25,24
   24 CONTINUE
      IF(ITER-NCE)26,25,26
   25 FSUM=FSUM+0.5*FY
      GO TO 29
   26 FSUM=FSUM+FY
   29 CONTINUE
C
C     PRINT SOLUTIONS
```

```
C
      IS = ITER/NJBP*NJBP
      IF(IS-ITER)31,33,31
   31 IF(ITER-1)32,33,32
   32 IF(ITER-NCE)34,33,34
   33 CONTINUE
      TAW = TAU+0.0001
      WRITE(3,203)TAW,VX,VY,X,Y,UX,UY,FY
   34 CONTINUE
   40 CONTINUE
C
C     DRAG COEFFICIENT OF CYLINDER DUE TO PARTICLE IMPACT
C
      PX=2.*FSUM*DELY
      WRITE(3,204)PX
      RETURN
C
C     FORMATS FOR OUTPUT STATEMENTS
C
  200 FORMAT( 65H1THE MOTION OF THE PARTICLES AT THE CYLINDER POSITION I
     1S GIVEN BY )
  201 FORMAT( 10HONZER   =   ,I3,20X, 10H NJBP   =   ,I3)
  202 FORMAT( 7HO    TAU, 11X, 4H VX , 12X, 4H VY , 11X, 4H   X , 12X,
     1     4H   Y , 14X, 2HUX, 14X, 2HUY , 14X, 4H FY /1H   )
  203 FORMAT ( 1H , F7.4, 4F16.6, 2F16.4, F16.5)
  204 FORMAT( 77HOTHE DRAG COEFFICIENT OF THE CYLINDER DUE TO THE REFLEC
     1TION OF PARTICLES IS  , F9.4 )
      END
```

THE DRAG ON CYLINDERS IN A STREAM OF DUST-LADEN AIR

THE PHYSICAL PARAMETERS ARE

```
XL     =  -20.000000
REZ    =   .100005E+03
XK     =   .100022E+02
P      =   .999874E+03

DC     =     2.53100
DP     =    .0010731
RHO    =     .001225
SIGMA  =    2.600000
XMU    =    .0001789
UF     =     13614.0
FSP    =     1013.25
FST    =      238.16
XMACH  =     .399995
```

IMPACTION EFFICIENCY OF A CIRCULAR CYLINDER

```
G4LEF  =     .000000
G4RIT  =    1.000000
SIGNL  =  -1.
DTAU   =     .100000
NIBP   =   10
NSBP   =    8
NX     =   20
```

THE MOTION OF A CRITICAL PARTICLE IS GIVEN BY

ITER = 1
G4LEF = .000000
G4ZER = .500000
G4RIT = 1.000000

TAU	G(1)	G(2)	G(3)	G(4)	UX	UY	CORE
.0000	.997078	.000124	-20.000000	.500000	.9971	.0001	24.0000
.8001	.997069	.000124	-19.202340	.500099	.9968	.0001	24.0669
1.6001	.997042	.000126	-18.404694	.500199	.9966	.0002	24.1475
2.4001	.996993	.000129	-17.607079	.500301	.9962	.0002	24.2338
3.2001	.996921	.000134	-16.809511	.500406	.9959	.0002	24.3278
4.0001	.996825	.000141	-16.012011	.500516	.9954	.0002	24.4318
4.8001	.996700	.000150	-15.214599	.500632	.9950	.0003	24.5486
5.6001	.996544	.000162	-14.417299	.500757	.9944	.0003	24.6818
6.4001	.996350	.000178	-13.620139	.500893	.9937	.0004	24.8363
7.2001	.996112	.000197	-12.823151	.501042	.9929	.0005	25.0184
8.0001	.995822	.000222	-12.026373	.501210	.9920	.0006	25.2364
8.8001	.995467	.000255	-11.229853	.501400	.9908	.0007	25.5017
9.6001	.995031	.000297	-10.433648	.501621	.9893	.0009	25.8296
10.4001	.994492	.000353	-9.637830	.501880	.9875	.0011	26.2411
11.2001	.993518	.000428	-8.842496	.502191	.9852	.0014	26.7642
12.0001	.992961	.000531	-8.047771	.502572	.9822	.0019	27.4338
12.8001	.991849	.000678	-7.253828	.503052	.9781	.0026	28.2849
13.6001	.990374	.000892	-6.460910	.503674	.9725	.0036	29.3259
14.4001	.988360	.001222	-5.669373	.504510	.9646	.0054	30.4886
15.2001	.985523	.001753	-4.879753	.505683	.9526	.0084	31.7494
16.0001	.981247	.002693	-4.092915	.507422	.9338	.0142	34.7849
16.8001	.974186	.004584	-3.310494	.510240	.9017	.0264	39.0942
17.6001	.961344	.009031	-2.535736	.515415	.8421	.0572	45.8068
18.4001	.934605	.022383	-1.775970	.526926	.7238	.1551	57.4493
19.2001	.873728	.080413	-1.049612	.562049	.5720	.5716	79.8745
19.4800	.846779	.141817	-.808958	.592148	.6836	.9280	96.4490
19.4830	.846582	.142772	-.806418	.592575	.6865	.9325	97.1067

THE MOTION OF A CRITICAL PARTICLE IS GIVEN BY

ITER = 10
G4LEF = .740234
G4ZER = .741211
G4RIT = .742188

TAU	G(1)	G(2)	G(3)	G(4)	UX	UY	CORE
.0000	.997083	.000183	-20.000000	.741211	.9971	.0002	24.0000
.8001	.997075	.000184	-19.202335	.741358	.9968	.0002	24.0668
1.6001	.997047	.000186	-18.404685	.741506	.9966	.0002	24.1472
2.4001	.996999	.000191	-17.607055	.741656	.9962	.0003	24.2335
3.2001	.996928	.000199	-16.809493	.741812	.9959	.0003	24.3273
4.0001	.996832	.000209	-16.011988	.741975	.9955	.0004	24.4310
4.8001	.996708	.000222	-15.214570	.742147	.9950	.0004	24.5475
5.6001	.996552	.000240	-14.417264	.742332	.9944	.0005	24.6804
6.4001	.996359	.000263	-13.620097	.742533	.9937	.0006	24.8344
7.2001	.996123	.000292	-12.823101	.742754	.9929	.0007	25.0159
8.0001	.995835	.000329	-12.026314	.743002	.9920	.0008	25.2330
8.8001	.995483	.000377	-11.229782	.743283	.9908	.0010	25.4970
9.6001	.995051	.000439	-10.433562	.743609	.9894	.0013	25.8231
10.4001	.994518	.000522	-9.637727	.743992	.9876	.0016	26.2320
11.2001	.993852	.000632	-8.842369	.744451	.9853	.0021	26.7514
12.0001	.993007	.000784	-8.047612	.745014	.9824	.0028	27.4156
12.8001	.991915	.000998	-7.253625	.745722	.9784	.0038	28.2591
13.6001	.990470	.001313	-6.460644	.746638	.9730	.0053	29.2914
14.4001	.988510	.001793	-5.669011	.747866	.9654	.0079	30.4469
15.2001	.985771	.002563	-4.879236	.749583	.9542	.0122	31.6872
16.0001	.981705	.003913	-4.092126	.752118	.9368	.0204	34.6120
16.8001	.975144	.006590	-3.309170	.756191	.9084	.0374	38.7749
17.6001	.963748	.012702	-2.533175	.763550	.8603	.0785	45.1259
18.4001	.942486	.029880	-1.769805	.779325	.7867	.1975	55.6653
19.2001	.909932	.091771	-1.028869	.822620	.8387	.5766	74.0378
19.9700	1.005195	.255842	-.313265	.952952	1.9787	.6491	98.9908
20.2620	1.135131	.265279	-.001020	1.031007	2.1855	.0021	99.2967

THE MOTION OF A CRITICAL PARTICLE IS GIVEN BY

ITER = 20
G4LEF = .740543
G4ZER = .740544
G4RIT = .740545

TAU	G(1)	C(2)	G(3)	G(4)	UX	UY	CDRE
.0000	.997083	.000183	-20.000000	.740544	.9971	.0002	24.0000
.8001	.997075	.000184	-19.202335	.740691	.9968	.0002	24.0668
1.6001	.997047	.000186	-18.404685	.740839	.9966	.0002	24.1472
2.4001	.996999	.000191	-17.607065	.740989	.9962	.0003	24.2335
3.2001	.996928	.000198	-16.809493	.741145	.9959	.0003	24.3273
4.0001	.996832	.000209	-16.011988	.741308	.9955	.0004	24.4310
4.8001	.996708	.000222	-15.214570	.741480	.9950	.0004	24.5475
5.6001	.996552	.000240	-14.417264	.741664	.9944	.0005	24.6804
6.4001	.996359	.000262	-13.620097	.741865	.9937	.0006	24.8344
7.2001	.996123	.000292	-12.823101	.742086	.9929	.0007	25.0159
8.0001	.995835	.000329	-12.026314	.742334	.9920	.0008	25.2330
8.8001	.995483	.000377	-11.229782	.742615	.9908	.0010	25.4970
9.6001	.995051	.000439	-10.433562	.742940	.9894	.0013	25.8231
10.4001	.994518	.000521	-9.637727	.743323	.9876	.0016	26.2321
11.2001	.993851	.000631	-8.842370	.743781	.9853	.0021	26.7514
12.0001	.993006	.000783	-8.047613	.744344	.9824	.0028	27.4156
12.8001	.991914	.000997	-7.253626	.745051	.9784	.0038	28.2592
13.6001	.990470	.001311	-6.460644	.745966	.9730	.0053	29.2915
14.4001	.988510	.001791	-5.669012	.747193	.9654	.0079	30.4470
15.2001	.985770	.002561	-4.879237	.748909	.9541	.0122	31.6874
16.0001	.981703	.003910	-4.092129	.751442	.9367	.0204	34.6125
16.8001	.975141	.006584	-3.309174	.755512	.9084	.0374	38.7759
17.6001	.963740	.012693	-2.533184	.762865	.8602	.0785	45.1280
18.4001	.942462	.029863	-1.769824	.778629	.7865	.1974	55.6706
19.2001	.909839	.091760	-1.028928	.821910	.8381	.5767	74.0532
19.9600	1.001089	.254359	-.323473	.949743	1.9612	.6665	98.9059
20.0320	1.031113	.264667	-.250330	.968453	2.0783	.5371	99.8149

THE IMPACTION EFFICIENCY IS .7405E+00

THE MOTION OF THE PARTICLES AT THE CYLINDER POSITION IS GIVEN BY

NZER = 120 NJBP = 4

TAU	VX	VY	X	Y	UX	UY	FY
19.2891	.796561	.000000	-1.000592	.000000	.0011	.0000	1.58846
19.2891	.796689	.005046	-1.000546	.021515	.0023	.0399	1.58776
19.2901	.796925	.011780	-.999543	.050206	.0052	.0930	1.58400
19.2921	.797326	.018529	-.997576	.078912	.0128	.1460	1.57718
19.2951	.797820	.025326	-.994646	.107643	.0224	.1989	1.56713
19.2991	.798460	.032183	-.990748	.136409	.0348	.2516	1.55395
19.3031	.799613	.038986	-.986680	.165181	.0515	.3034	1.53837
19.3081	.800893	.045857	-.981640	.193995	.0712	.3546	1.51959
19.3141	.805018	.050977	-.975503	.222780	.0937	.4053	1.50357
19.3211	.806501	.057816	-.968497	.251687	.1196	.4553	1.47832
19.3291	.808174	.064786	-.960505	.280663	.1487	.5046	1.44987
19.3381	.810123	.071838	-.951518	.309718	.1812	.5529	1.41835
19.3481	.812352	.078997	-.941531	.338861	.2172	.6001	1.38371
19.3591	.814894	.086261	-.930533	.368102	.2568	.6460	1.34596
19.3711	.817799	.093587	-.918514	.397450	.3002	.6904	1.30513
19.3851	.820877	.101292	-.904641	.427015	.3474	.7345	1.26047
19.3991	.824591	.108818	-.890539	.456610	.3987	.7750	1.21337
19.4151	.828612	.116637	-.874548	.486453	.4543	.8145	1.16251
19.4331	.833249	.121845	-.856623	.516382	.5144	.8527	1.11111
19.4521	.838067	.130146	-.837612	.546628	.5795	.8874	1.05307
19.4721	.843566	.138438	-.817470	.577055	.6491	.9180	.99194
19.4951	.849679	.147311	-.794472	.607960	.7253	.9469	.92632
19.5201	.856633	.156406	-.769409	.639238	.8075	.9713	.85690
19.5471	.863130	.162873	-.742334	.670693	.8955	.9903	.78539
19.5771	.872059	.172701	-.712147	.702898	.9915	1.0035	.70765
19.6111	.882457	.183118	-.677922	.735924	1.0971	1.0097	.62475
19.6501	.892168	.192024	-.638832	.769850	1.2137	1.0066	.53655
19.6951	.907054	.204100	-.593431	.805268	1.3439	.9900	.44155
19.7491	.925827	.217246	-.538722	.842827	1.4923	.9536	.33824
19.8191	.948130	.231167	-.467963	.884139	1.6686	.8829	.22231
19.9371	.990842	.251987	-.347991	.937792	1.9213	.7128	.07457
20.0321	1.031113	.264667	-.250330	.968453	2.0783	.5371	.00032

THE DRAG COEFFICIENT OF THE CYLINDER DUE TO THE REFLECTION OF PARTICLES IS 1.6540

THE TARGET REYNOLDS NUMBER IS .2359E+06

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ESTIMATION OF EXTRA DRAG FORCE ON A CIRCULAR CYLINDER DUE TO THE PRESENCE OF SOLID PARTICLES IN SUBSONIC COMPRESSIBLE FLOW

4. AUTHORS (Last name, first name, middle initial. If military, show rank, e.g. Doe, Maj. John E.)

MELLSEN, STANLEY B.

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The effect of particles, such as dust in air, on aerocynamic drag of circular cylinders was calculated for compressible flow at critical Mach number and for incompressible flow. The effect of compressibility was found negligible for particles larger than about 10μm, for which the air can be considered a continuum. Drag coefficient and collection efficiency are provided for a wide range of inertia parameters and Reynolds numbers for both compressible and incompressible flow.

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PARTICULATE IMPACTION
DUSTY COMPRESSIBLE FLOW
BLAST WAVE